# Fractal Aesthetics in Geometrical Art Forms

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## Abstract

Not being able to "rationalize" all algebraic-geometric shapes found in Nature provides evidence that even non-Euclidean geometries are not enough to describe complex natural phenomena and shapes, unless we introduce "fractional sizes" (where one can see properties and qualities of algebraic-geometric shapes that Nature offers to our perception). "Fractal Geometry", that is based on "new dimensions" and makes use of recursive dynamical systems, has become a new "customized" form of "Dynamical Art" made peculiar by new codes and patterns dictated by "Fractals". The XX Century is recognized as a period marked by many new forms of Art and languages which overlap onto each other, from Expressionism to Cubism, Surrealism, Constructivism, Futurism, and so on; each one characterized by a "code" that can be adopted to distinguish an Art movement from another. The "aesthetic beauty of Fractals" is a "mathematically minded" artistic code that helps the creativity of the artist.

## 1 Fractal Aesthetics and Algebraic Constructions

The term "Fractal" (from the Latin "interrupted" or "fragmented") was introduced in Mathematics in the second half of the XX Century by B. Mandelbrot, who defined so-called "Fractal Geometry" [1],[2]. This new Geometry, freeing itself from older stereotypes, is able to describe and justify many "irregular although beautiful" shapes found and perceptible in Nature, taking advantage of such notions as: "fractional dimensional indices" and the characterizing property called "self-similarity" [3]. Shortly speaking, Fractal Geometry is based on the use of the "principle of repetition" of geometric shapes, so that an object (e.g., a subset of space or an hyper-surface) retains and self-reproduces its shape when subjected to any degree of magnification. In a sense, "self-similarity" corresponds to verify a suitable "property of internal homothety". With so-called "String Theory" [4] Physicists have speculated in the seventies around a Universe with at least eleven dimensions (where besides the usual "geometrical dimensions" of SpaceTime there are extra dimensions able to include Gravity, Time, Electromagnetism, Weak Nuclear Forces, and so on); while mathematicians, working with "Fractals", have envisaged areas described by "limit curves" that are more than 1-dimensional, so that the direction of travel may change infinitely many times. Fractals find applications even in understanding the structure of Kosmos [5]. With the new Geometry of Mandelbrot Art has received a "liberating impulse" towards what is usually called the "artistic creativity". The passage from the Euclidean Space to Riemannian Spaces (typical of Cubism and Futurism; [6]) in "Fractal Art" seems to be overcome, or at least partly repudiated, by means of new and original models through which the artist can express "reality". The models proposed by "Fractal Art" are no longer caught in a given number of dimensions, since the same Fractals have neither a form nor well-defined spaces ([7], from which we borrow some ideas). In this new way of understanding forms, Art becomes an "epistemological thermometer" through images, sounds, colors; it carves out a role that goes beyond the "pure" scientific aspects of Fractal Geometry itself, by using representative systems of non-linearity dictated by a new understanding on perceptive properties of objects. Fractal Art proposes again "Infinity", although in a systematic but discontinuous approach to the same "forms" existing before. Such an attitude allows artists to discover that "sense of infinity" which Kant claimed to be one of the feelings that *"stir the human soul"*. Fractal images created by Mandelbrot algorithms arouse an interest in Art and Science even greater than his theory by itself; with the advent of new software, the intertwining between Art and Science has become increasingly stronger [8]. This eventually generated new models dictated by computational complexity of mathematically recursive techniques, with the aim to represent and write "codes" that could (and should) reveal some of the "secrets of Nature". Mandelbrot himself said once: *"It is believed that somehow there exists a fractal correspondence with the structure of the human mind, that is why people find them so familiar. This familiarity is still a mystery and deepens the argument more than the mystery of growing*." Fractal Geometry becomes thence "*a way of description, calculation and reflection on the forms that are irregular and fragmented, jagged and broken curves crystalline snowflakes dust staple of galaxies"* ([9], p. 117). Mandelbrot's work returned in fact onto previous mathematical studies on curves due mathematicians of XIX Century who had already produced examples of "Fractals" that, because of their "abstruse properties", were considered "pathological." At the same time, mathematicians like F. Hausdorff and A. Besicovitch redefined the "size of the space" for the representations of these curves known as "pathological", so to include what emerged to be a notion of "fractional dimensions". Fractal Geometry, was thus offering artists the ability to "restructure reality" and to highlight aspects that were previously "neglected", through new explorative mechanisms dictated by "irregular harmonies". "Mathematical" Artists such as R. Voss, J. Clarke, G. Turk and A. Norton, who suitably manipulated algorithms of Mandelbrot, gave thence birth to a lot of complex fractal shapes that found applications in several fields of imagery, both in Art and Science in general.

With "Fractal Art" older paradigms seem to be wiped away. Perspective (whose origins can be even traced back to studies on Geometrical Optics in the Hellenistic period and that was strongly proposed by Filippo Brunelleschi, Leon Battista Alberti and Luca Pacioli), the "anamorphosis", the Golden Mean of Leonardo da Vinci and Fibonacci, the fourth dimension in Salvador Dalì seem all to become "obsolete stereotypes". Fractal Art is a unique form of creativity where the complexity of Art forms turns out to be an instrument linked to the structure and the "geometry" of the curves (and surfaces) generated; it is further enhanced by the effect that "the emergence of order from chaos" conveys at the emotional level and also by the clever choice of colors that accompany the fractal shapes. Many works of Futurist painters and "action painting" measures can be classified among "Fractal Art" [11],[12]. Among the greatest exponents of Fractal Art we mention J. Pollock. W. Kandinsky, P. Klee, P. Mondriaan, M.C. Escher, N. Chlebnikowski, S. Dalì, A. D'Anna, G. Balla, M. Duchamp, as well as many others [8]. Jackson Pollock (1912-1956) was the founder of "action painting", characterized by a special technique called "dripping" (see also [13]). He said *"when I paint I do not have the correct perception of what is happening, until I realize what I did ... "* In fact, according to the physicist Richard P. Taylor, he painted fractals twenty-five years before their "official" discovery in natural phenomena. This is what happened according to a careful and thorough analysis on the computer; according to Taylor a chronological study of Pollock's paintings shows that Pollock continuously refined his technique. The analysis revealed these increasing "fractal complexity" of fractal patterns in the artworks [14]. Science, in fact, until the sixties had always believed that Nature operates in a disordered way; this viewpoint generated the theory of "random chaos". Only with the advent of Fractal Geometry, Benoit Mandelbrot was eventually able to claim that "*the clouds are not spheres, mountains are not cones, coastlines are not circles, the tree bark is not smooth, and lightning does not travel in a straight line*" [7]. Being able to play both with the infinitely large and the infinitely small has given an important new breathing to Art - contrary to what Hegel said once about the death of art-redefining creativity and artistic perspective more than in any other sector of human Culture. We come therefore to see that Nature manifests his "Art" with a *"regular irregularity"* (implicit in the scale invariance properties of self-similarity, for which the type and degree of irregularity of the object remains identical at nested scales of construction and perception). This new Geometry better shows, in a sense, the relationship existing between the object and the "scaled position" of the observer.

With the aid of a computer it is relatively easy to generate new Fractals, where - because of their complexity - one can recognize the law of chaos, now understood as a normal manifestation of Nature within the figure itself. This does not only mean that within a fractal structure there is *"hidden symmetry"*, indeed hidden in the property of self-similarity, but more than this one can recognize a new concept of *"symmetry and harmony"* that suitable extends the older notions of classical Greek theory of *"harmony and proportions"*.

## 2 Fractals in Mathematics

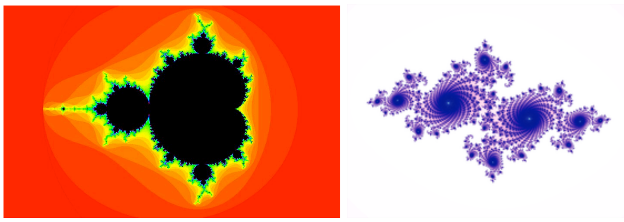

Fig. 1 – *Mandelbrot and Julia Set*

The Mandelbrot set (see Figure 1) is described by the equation $\Gamma(\phi) = \phi_p^2 + C_0$ where $\phi_p$ and $C_0$ are numbers in the complex plane, and $\phi_0 = 0$. The sequence $\phi_{p+1} = \phi_p^2 + C_0$ may be convergent in the complex plane and tends to infinity only if $|\phi_p| > 2$, thus introducing sets that may be connected or not connected. The term $C_0$ is called the "control parameter" and its choice may be arbitrary. The "fractal dimension" defines the degree of irregularity of the Fractal; there are several definitions of "fractal dimension": one of the most appropriate is known as the "Hausdorff dimension" [3]. Fractals can be represented by a first order equation (usually called "linear") or generated by an equation of higher degree (these are called "non-linear"). A "quadratic Fractal" was described in 1918 by G. Julia and P. Fatou, who based his analysis on the behavior of the transformations of the following equation $g(z) = z^2 + c$ and proved the following result (used later to create those images that are nowadays called " figures of Julia"): *Set in the Complex Numbers the recurrence* $z_{n+1} = z_n^2 + c$, *with c a "control parameter". Let* $r(c) = max(2, |c|)$. *Then if* $|z_0| > r(c)$, *the initial point* $z_0$ *tends to infinity, otherwise the sequence converges to a finite point in the plane. The set Jc of those initial points that do not tend to infinity is called the Julia set of the point c.* Julia figures can only be drawn with the aid of a computer; see Fig. 1). The set of all values *c* for which the Julia set is connected is known as the "Mandelbrot set".

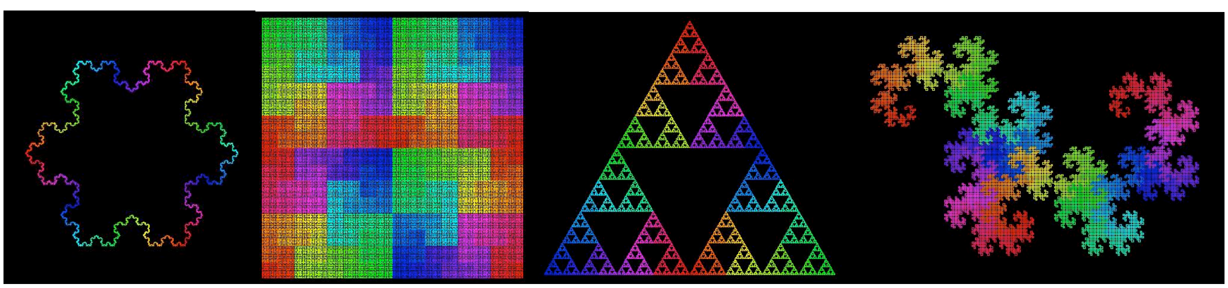

Fig. 2 – *a) Koch Curve; b) Hilbert Curve; c) Sierpinski Curve; d) Dragon's Curve*

Ultimately there are "generative fractals" used to describe more or less smooth surfaces and to build more sophisticated models; e.g., the so-called "Sierpinski Fractals" [15] - that in their "pre-fractal form" can be found in XII Century Art [16] - obtained from an equilateral triangle or other regular polygons and polyhedra. An important work in the field of Fractals is used in Physics under the name of "electrostatic potential of the Mandelbrot set" [17]. Fractals are in fact generated by dynamical systems that, under suitable conditions (namely, in the presence of either external perturbations of dissipative effects) may present an even richer chaotic behavior. In particular, they give rise to the so-called "strange attractors" (like the famous "butterfly effect" discovered by the American mathematician E. Lorentz - Fig. 3) that are characterized by a great sensitivity to the initial conditions. The description of a Fractal cannot be just bound to an algorithm together with its computational complexity. It is also related to the perception and the reproduction of the fractal image itself, that can be provided to us either by Nature, by an artistic process of iterated construction (as in Pollock's paintings) or by Digital Art through a computer-generated object, or even through other Digital techniques (such as, e.g., the "Painting with Light" techniques of

Digital Photography described in [11]).

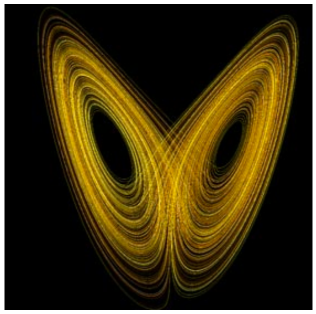

Fig. 3 – *Lorentz Butterfly*

## 3 Conclusions

The discovery of Fractal Geometry has allowed to go beyond the "continuum Geometries" of Euclid and Riemann, through a theory of "complex geometric shapes" that do not meet conditions of continuity and differentiability but are still "pleasant". Fractals allow to better understand geometric figures with "irregular infinitesimal structures", somehow realizing Einstein's predictions about non-linear phenomena in the Universe. Fractal Geometry, playing with the infinitely large and the infinitely small through digital simulations, has gradually incorporated images involving all forms of Art and Science. Even the architecture of many churches at various times has a fractal structure. Famous examples are the Gothic cathedrals, in which basic structures are repeated at different scales of self-similarity, showing (pre)-fractal properties. Mandelbrot quoted the example of the Paris Opera, a building with a "scale symmetry". Walking along Rue de l'Opera, the closer you get the more you notice fractalized details. Henry Poincaré, one of the proponents of chaos theory in XIX century, argued: *"When a scientist worthy of the name, and above all a mathematician, works on the same impression of an artist, he gives to his work a joy just as great and of the same nature"* [18]. Therefore Science and Art cannot have an autonomous behavior.

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