Mathematical Modeling of Cartoon Images and Other Objects

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Abstract

Graphs of functions and other mathematical objects can be easily manipulated with the aim of producing mathematical models for cartoon images or other images. Experience showed that this can be an excellent pedagogical tool to approach several subjects in basic and high school mathematics. This is a hands-on workshop where participants will be faced with different activities that can be used in classroom. Although any dynamic geometric computer program can be used for this propose, the activities in the workshop will be done using GeoGebra.

1. Introduction

Mathematical objects such as functions or geometric figures can be used to produce mathematical models for a naïve representation of reality. In [1] we described a pedagogical project to create cartoon images with pieces of graphs of functions. This project was intended to help high school students to study functions, but it raised the interest of younger students, even not yet knowing functions. So we enlarged the scope of the project and we started using images composed of geometric objects to teach or motivate concepts such as point coordinates, symmetry or translation.

In this workshop we intend to lead the participants to figure out how powerful this approach can be to teach different subjects in school mathematics or, simply, to keep students motivated as they are given tasks they can perform, even if they are not good math students. Different activities will be proposed to participants, who will be invited to perform them. Pedagogical issues related to each activity will be highlighted and discussed.

Although any dynamic geometric computer program can be used, the activities in the workshop will be done using GeoGebra. This choice was based in four of its greatest qualities: it is free and user-friendly; it does not require a lot of computer-space and produces small files.

2. Using Functions

In [1] the idea of using functions to model cartoon images was presented, and several tasks concerned to it were suggested to be used in classroom. The key idea behind the entire project is always keeping together the image and the recipe that produces it, so that students can really learn mathematics. And how can this be achieved?

Let's take the example of the boat in figure 1. It is easy for students to understand that the sea is modeled by the function $y=sin(x)$ (or $y=cos(x)$), but they also understand that if they want to produce nice waves it is necessary to manipulate the initial function and so, in a practical way and with a concrete goal, they can learn a lot about the effects of adding or multiplying constants to the expression of a function.

To sketch the boat we simply use parabolas. Again students have to manipulate the coefficients of $y=ax^2+bx+c$ to produce the different parabolas that, together, form the boat. It is clear that to make this kind of drawings the notion of restriction of a function is indispensable and that, to define the domain for the restrictions needed, often students have to find the points of intersection of two curves. Depending on what skills we want them to acquire, we can ask them to do these tasks manually or using the facilities of a dynamic geometric computer program. One way or the other, the result is always positive. Students are enthusiastic and start talking of domains and restrictions of functions as something familiar.

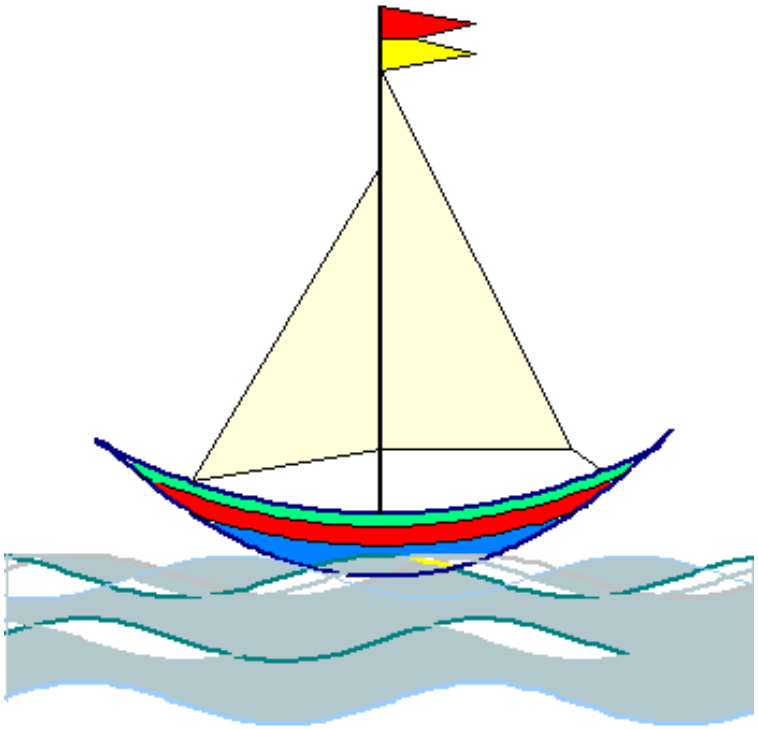

Figure 1: *Boat at a wavy sea.*

3. Using Geometric Objects

For younger students, who have not yet made acquaintance with the concept of function, the initial project was adapted to create images using geometric objects such as points, segments of straight lines, polygons, circles, ellipses, etc.. Many nice and simple drawings can be done with these objects. Keeping always in mind that each drawing has an associated mathematical recipe, inviting students to deal with these drawings helps them to learn, not only a lot about the shapes involved, but also how to relate an image to the way it can be formally defined in a Cartesian coordinate system. Many different activities can be proposed, depending on the educational objectives. Following a "recipe", students can just start by writing point coordinates in the plane and, then, use them to define polygons or other shapes that, together, give raise to a concrete image. Pine tree in figure 2 is an example of the procedure we just described.

Figure 2: *The pine tree*

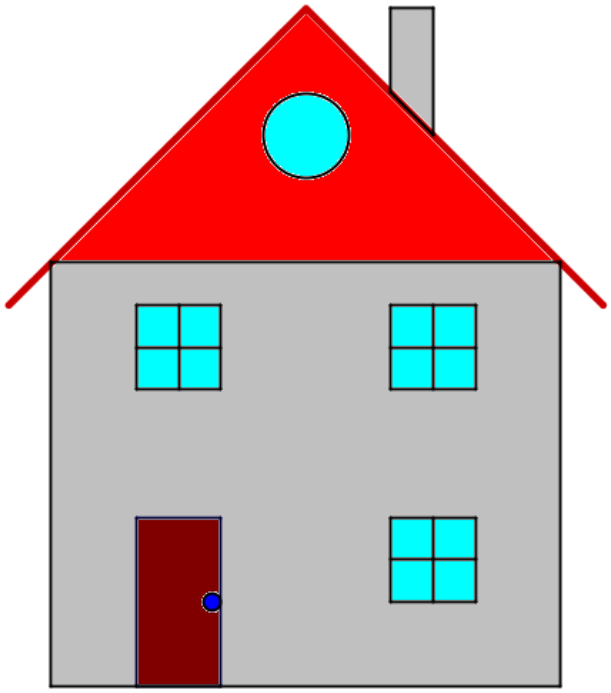

Figure 3: *The house*

We had primary school students performing this task, even before they had learned point coordinates – for sure at the end they knew a lot about it.

In the following section we describe other possible activities, based on our experience in classroom.

4. Classroom activities

All the experiences of applying this project in classroom were successful. Usually the first activity proposed to students consists on giving them a mathematical recipe – with functions or geometrical objects, according to their level – and asking them to make the corresponding drawing. This is an easy task but it is important for them to start relating shapes and their mathematical expressions and also to get used to the computer program. For younger students "the house" (figure 3) is a good starting point.

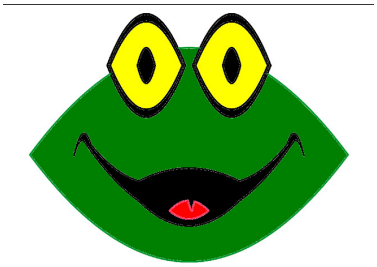

Figure 4: *Frog*

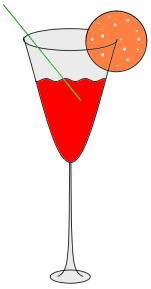

Figure 5: *Tropical drink*

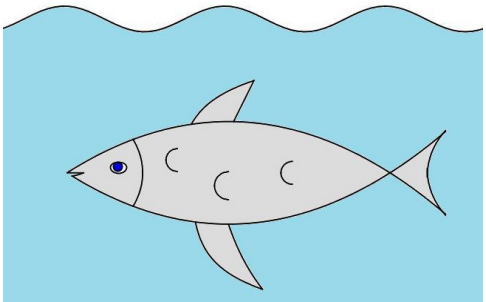

Figure 6: *Tuna fish.*

For high school students the departure point can be different according to the grade they attend. For the beginners, the frog in figure 4, involving only quadratic functions, can be a good choice. This frog

image is very popular in Portugal, as it is the logo for a well known web service. For second year students it is good to give them a recipe that includes trigonometric functions. For this we have different offers, some very simple, as the boat in figure 1, or a more elaborated as the tropical drink in figure 5. For final year students it is interesting that they discover that graphs of logarithmic or exponential functions can also model some images. For this propose one possible choice is the tuna fish in figure 6.

After this first step, students generally become very excited and they start creating their own drawings with no need for a recipe, or simply decide to improve the initial recipes. In figure 8 it is possible to see the house after some improvements on the roof and windows done by the group of students in figure 7.

Figure 7: *Group of students following the recipe for the house in figure 3*

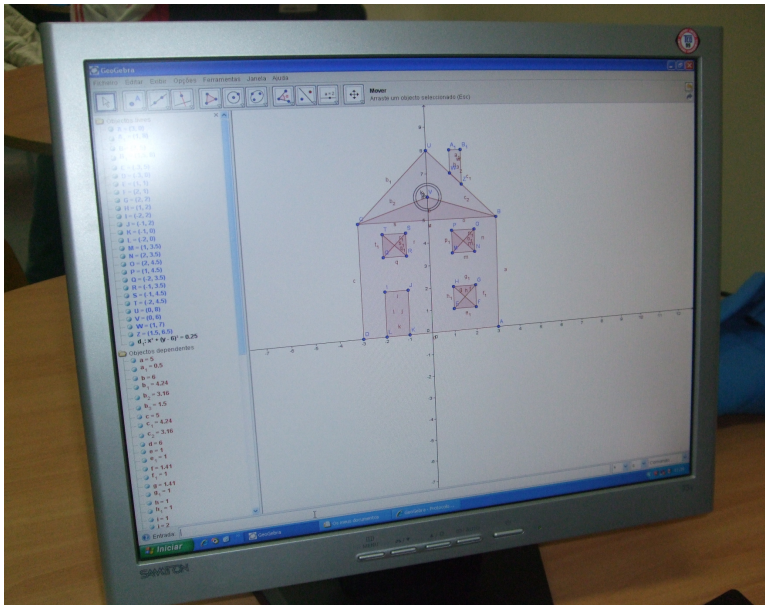

Figure 8: *House with improvements*

Once students are familiar with the idea, the activities can become gradually more directed to specific proposes. Sometimes all that is needed to motivate students and free their creativity is to ask them to create their own image and corresponding recipe.

One very interesting fact is that even students that claim not to like mathematics, and generally are not very cooperative in classroom, after one or two activities became the most enthusiastic about the tasks we give them. Besides the usual comment "This is fun", student comments usually refer to the fact that they understand better the concepts because they can visualize them.

Teachers are often astonished by the behavior of their students that suddenly become interested and cooperative in classroom, and start talking about math subjects in a familiar way. Teachers often ask us to go to schools to provide training to students or to other teachers.

To keep the students' interest high, activities have to be carefully designed, and the topics where this idea can be used have also to be carefully chosen. For example, geometric transformations are a good source of inspiration for such activities. Students can understand more deeply the concepts of reflection or translation if they use them in their drawings. To study translations a simple, but effective task consists of transforming a pine tree in a pinewood. We challenge the students to apply translations associated to different vectors to the pine tree of figure 2 and they get something similar to the pinewood shown in figure 9. Of course they have to carefully write down the vectors used, because they are asked to write a recipe that can be used by other colleagues.

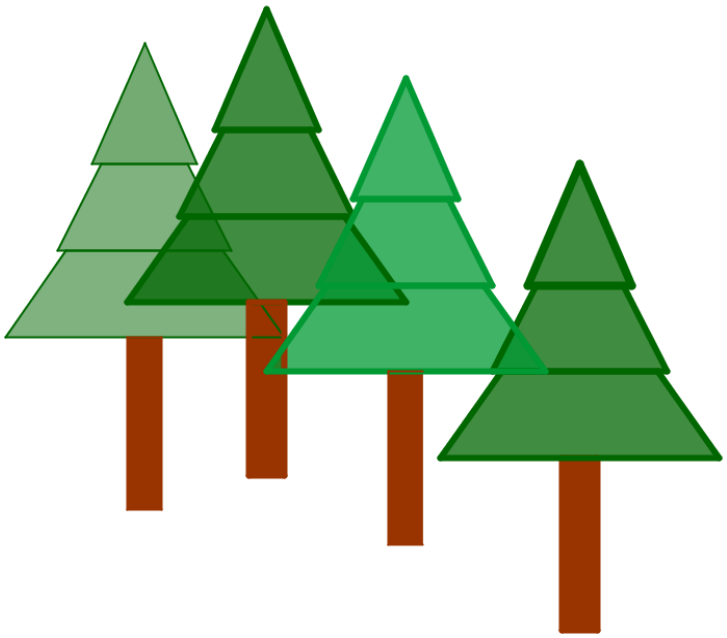

Figure 9: *Pinewood*

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Another successful task is to give them the recipe for a "half-house" as in figure 10. When they discover that the recipe does not provide a complete house we ask them to complete both the drawing and the recipe to get a complete house, using a reflection on the axis Oy. We want them to associate to each point drawn by symmetry the corresponding coordinates. If they finish this task easily it means that they already got the notion of reflection and then we ask them to create their own "half-recipes" for "half-objects" that can be completed by symmetry with respect to one of the coordinate axes or, even, to another straight line.

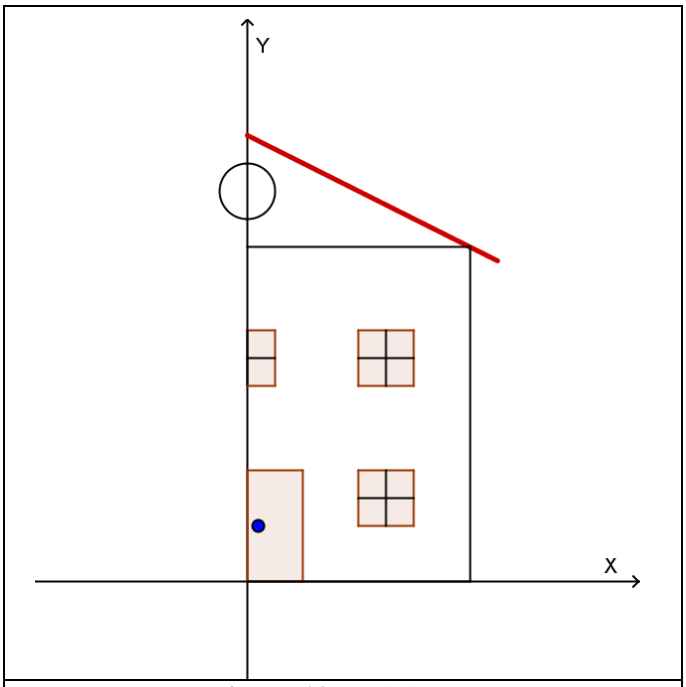

Figure 10: *Half-house*

5. Activities in the Workshop

The computer program we will use, GeoGebra, is free and can be easily downloaded from www.geogebra.org/cms/. It is very user friendly and no previous knowledge of it is required. Participants in the workshop can enjoy it more if they bring their own laptops with GeoGebra already installed.

Participants don't have to be experts in mathematics and previous knowledge of graphs of elementary functions is recommended but it is not essential. However, this workshop is mainly directed at teachers of mathematics at various levels.

We will begin with a brief explanation about the program, with particular reference to the use of parameters that can be dynamically adjusted to modify appropriately a mathematical object. Next, the same way we usually do with students, we will propose to the participants just to follow a mathematical recipe and discover what the corresponding image is. This can be done alone or in groups and there will be different recipes for each group. According to the performance and the interests of each group other tasks will be proposed, including the ones described above. Participants will be encouraged to discuss the given tasks. All suggestions, comments, new ideas will be very welcome.

By the end of the workshop, in addition to being convinced that this is a good teaching tool, we hope that the participants had had lots of fun and can now use mathematics to create their own cartoon images.

We wish to thank the authors of the images in pictures 4, 5 and 6, Filipa Lecoq, Jorge Pinto e José Afonso Martins.

References

[1] M.G. Marques and M. Pires, "Creating Cartoon Images with Functions: A Pedagogical Project", in G.W. Hart and R. Sarhangi (eds) *Bridges Pécs: Mathematics, Music, Art, Architecture, Culture*, pp 435-438. 2010