Creative Circle Design

Susan McBurney
Western Springs, IL 60558, USA
E-mail: smcburne@iit.edu

Abstract

The process of creating with circles has intrigued geometers and artisans since ancient times. This article will briefly examine a variety of designs involving circles from ancient to modern times and their applications, ranging from geometric proof to ornamentation.

Early Circle Use

Curved elements have a particular aesthetic appeal and have been utilized by artists across cultures in a wide variety of ways. It could even be argued that the first non-linear design element ever was created when some anonymous caveperson drew in the sand or mud with a stick while turning 360°. In any case, that was just the beginning of a long line of practical, as well as decorative applications.

For example in his book, "The Mathematics of the Heavens and the Earth", [1], Van Brummelen comments that "much ancient and medieval trigonometry used circles and their arcs rather than triangles as their reference figures." It is also interesting to note the appearance of the circle in geometric proofs where it would not be necessarily expected. For example, the square root of any number can be found using overlapping circles within a circumscribed circle. An example is given for finding the square root of three Figure 1.

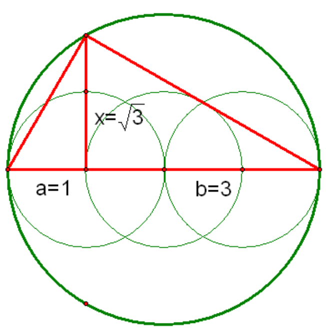

Figure 1: *Finding the square root*

Because an angle inscribed in a semi-circle is a right angle, the triangles in Figure 1 are all similar. Let a = 1 and b = 3. Then, by similar triangles,

$$\frac{a}{x}=\frac{x}{b}, \quad x^2=ab, \quad x=\sqrt{ab}$$

That is, the vertical line, x, is equal in length to the square root of 3. Any number can be substituted for b by using the appropriate number of circles and setting the first radius equal to one.

Guilloches and Solomon's Knots

One classic circular design that has been widely used across cultures is the guilloche, pictured in Figure 2. Defined by Webster as "An ornamental design formed of loosely interlaced bands, the openings being filled with round ornaments", it has many variations of form. Figures 2 and 3 show a modern example along side an Assyrian design of 7th century B.C.

Figure 2: *A modern guilloche design*

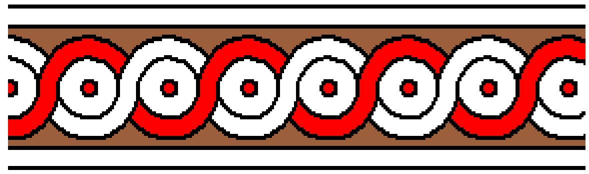

Figure 3: *An Assyrian band, 7th c. B.C.*

A circular guilloche is known as a Solomon's Knot and it, too, has been widely used throughout the centuries. It can be constructed with any desired number of circles and band widths as illustrated in Fig. 4.

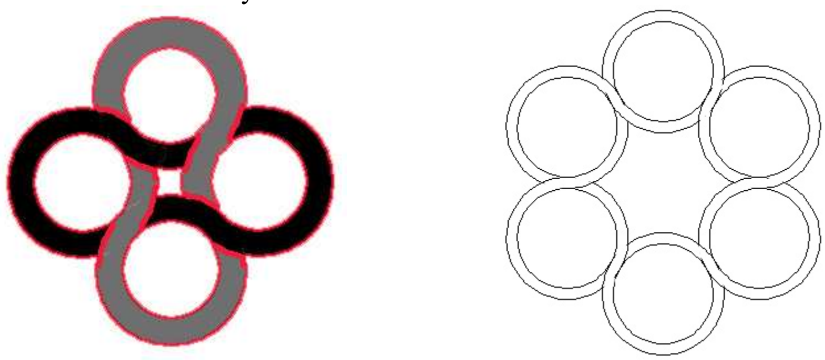

Figure 4: *Two versions of Solomon's Knot*

A generalized n-circle guilloche can be constructed in the following manner [4].

1. Inscribe n circles within an outer circle such that all are tangent to each other. See Figure 5. Do this by dividing the circle into n equal parts, either geometrically or by measuring. See Figure 6. Bisect the angle θ. Construct a tangent at the point where the bisector crosses the outer circle. Draw a radius to the tangent, find the midpoint, draw a circle. Repeat n-1 times.
2. Draw larger and smaller circles as desired to create a band Figure 7.
3. Erase appropriate lines to create desired interweaving of the bands.

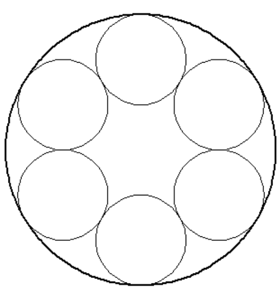

Figure 5: *n = 6*

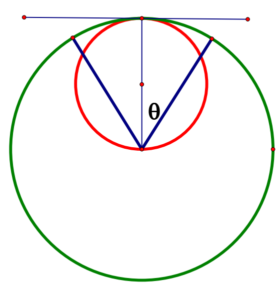

Figure 6: *Position the first circle*

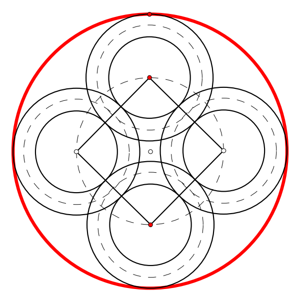

Figure 7: *n = 4*

DaVinci's Lunes

Leonardo DaVinci (1452-1519) developed an interest in geometry somewhat later in life, but then pursued it systematically until his death. He was particularly interested in applications of lunes and he left two beautiful notebook pages fill with more than 170 semi-circular designs, his "lunulae". Certain sections were filled such that it appears he was showing the conservation of area among each of the designs. In each diagram the filled sections have the same total area.

Many of the diagrams were based on the frame in Figure 8. The two gray sections have equal areas. This can be proved with the following logic. Let the radius of the outside circle equal one, then the area of the whole circle will be π Figure 9. The area of an interior circle with a radius of 1/2 will be π/4 Figure 10. The top half of the interior circle has area π/8 Figure 11, but so does the gray section in Figure 12 since the angle of the gray sector is 45°. The only differences in these two equal areas are shown in Figure 8. That is, the two gray areas must be equal.

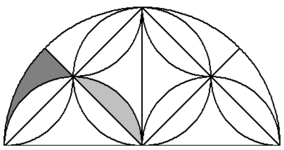

Figure 8: *Gray areas are equal*

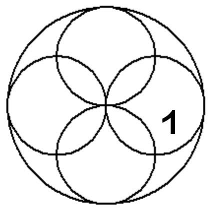

Figure 9: *Area of large circle = π*

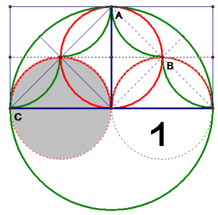

Figure 10*: Area equals π/4*

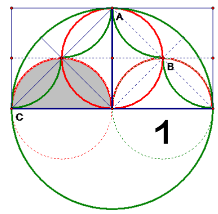

Figure 11: *Area equals π/8*

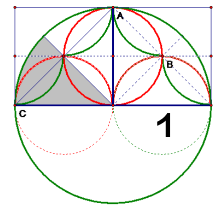

Figure 12: *Area equals π/8*

Now one can see there are eight equal areas on a side, six of them "almond-shaped" and two "hook-shaped". Various combinations of four at a time can be filled in. The number of possible combinations would be ${}_8C_4$, or 70. The author generated tables of all 70 possible bi-symmetric designs and Figure 13 shows just four of them. DaVinci included these four in his diagrams and he had others as well, but not all of the seventy. However, there are many other equal-area arrangements which he did include. Some of these were addressed in [7] while others are the subject of ongoing research.

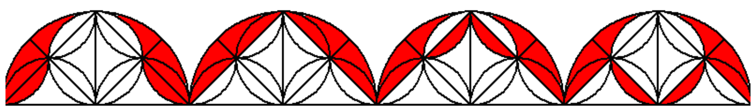

Figure 13: *Four of the 70 possible combinations. All filled areas are equal.*

The Area of a Sector

The area of a segment of a circle can be obtained from the size of the angle θ which subtends it. The exact value is given by $A = \frac{r^2}{2}\left(\frac{\pi\theta}{180} - \sin\theta\right)$. In Figure 14 where the angle 90° forms the light gray segment and the radius is 1/2, the area would equal 1/8 (pi/2 – 1) ≈ 0.07135 square units. This could be referred to as the Basic Unit, or BU since it forms the basis of all the previous designs. In Figure 15 the radius of the circle is $r = \frac{\sqrt{2}}{2}$ and the area of the segment is 0.1427 square units, equal to two BUs. DaVinci used this relationship in his diagram shown in Figure 16. Again, equal areas are filled in.

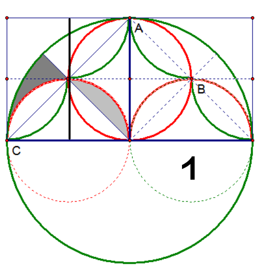

Figure 14: *Basic Unit*

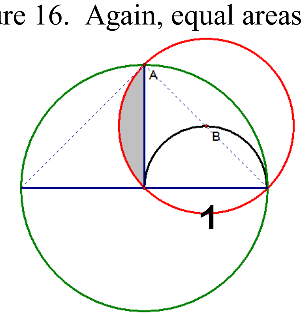

Figure 15: *Two Basic units*

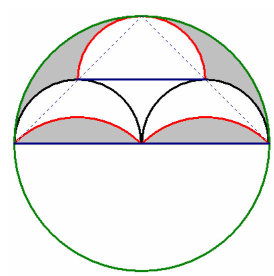

Figure 16: *Larger units*

Later Use of Circular Design

The creative use of circles spans many centuries and is embedded in many cultures. Only a brief introduction can be shown here although there are many other uses that deserve to be included as well. Two that show especially strong links to both geometry and art are the intricate 8th c. Celtic scrollwork and the later beautiful tracery of medieval Gothic architecture.

Two of the greatest American architects of the 19th and 20th centuries also employed circles in their work. Louis Sullivan (1856-1924) was known for his florid ornamental style as well as his architectural projects. One of his most iconic designs was the circular elevator grill he designed for the Chicago Stock Exchange Building, 1893-4. See Figure 17.

Frank Lloyd Wright (1867-1959) was of course known for his innovative Prairie Style architecture which emphasized modern, horizontal lines. He often drew upon geometric elements, however, for the decorative details of his designs. A lovely 1912 window of his is shown in Figure 18. It was designed for the Avery Coonley Playhouse in Riverside, IL, and features brightly colored circles meant to represent balloons.

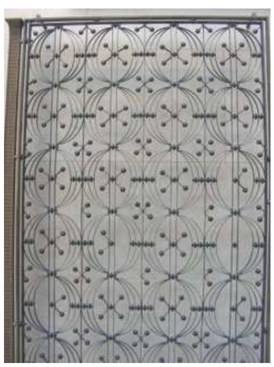

Figure 17: *Elevator Grill by Louis Sullivan (1894)*

Figure 18: *Playhouse window by Frank Lloyd Wright (1912)*

It is interesting and instructive to trace the use of a particular design element through time and across different cultures. The circle is surely one of the most basic units, yet it has been applied both decoratively and practically in a wide variety of ways. Some cultures seem to have used it very little while others applied it extensively. Purely utilitarian applications range from geometric proofs to building ornamentation. Technical advances also play a part. As new technologies become available, adventurous designers can use them to expand their scope of creativity. In the end, it is the ingenuity of the artist, architect, or mathematician that contributes the most to the beautiful utilization of the circle.

References

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