A Virtual Installation of Sierpinski Triangle

Mehrdad Garousi
Freelance fractal artist
No. 153, Second floor, Block #14
Maskan Apartments, Kashani Ave
Hamadan, Iran
E-mail: mehrdad_fractal@yahoo.com
http://mehrdadart.deviantart.com

Hamed Akbari
Islamic Azad University, Hamadan Branch
Hamadan, Iran
E-mail: hamedakbari2001@yahoo.com

Abstract

Discovering some kind of illusive three dimensional perspective lain in a certain two dimensional construction of Sierpinski Triangle, we will have a three-dimensional-like especial digital installation artwork of this fractal on a two dimensional screen while, like other fractals, it basically retains its fractional dimension.

Introduction

Retroactive to the previous paper in yesteryear's Bridges [1], describing the construction of a Sierpinski triangle with a generator of three intersected circles, this paper discloses a more interesting aspect of this always-mysterious fractal. Here, we start with a perfect circular ring as our initiator (Figure 1) and put three other smaller and completely distinct rings as our generator under it. This way, we will have our first iteration completed in Figure 2. To have a more artistic view, the initiator of the process that is a single large ring is omitted in all other next iterations. Needless to say that the ring-like circles used as constructive units should be assumed to be 2D flat circles and they are shown as rings only in order to have a better exposure of different iterations and clearer perception of next occurring intersections and overlays.

The generator consists of a triadic set of similar circles with mild and equal distances from each other. Diameter of these circles each is equal to half of the diameter of the initiator. As it is apparent, like any other way of constructing Sierpinski triangles, we repeat placing our generator consecutively in any constructive units of certain iterations to generate next iterations of 2 (Figure 3), 3 (Figure 4), 4 (Figure 5), and 10 (Figure 6). Eventually, decreasing the thickness of rings and increasing their number of repetition, Figure 7 with 20 iterations presents a better and more sensible view of our goal fractal, Sierpinski triangle.

The number of the rings in every iteration, condoning the presence of the first large initiator circle which is omitted intentionally in next iterations, can be calculated according to the formula: N(number of rings) = $(3^n+3^{n-1}+...3)$ in which n is the respective number of each certain iteration.

As it is seen, our uniform fractal is constructed of distinct circles and the free distances between them are removed during iterations. Actually, because constructing function is contractive, separate triadic circles of every iteration always tend to bring points closer together and reach each other at higher iterations [2]. Having a closer look at tendency points (Figure 8) will launch further comprehensions. This kind of transcend of tunnel-like set of circles towards the infinity reminds of the convergence of parallel borderlines of a road or tunnel in reality. Specially, no matter we magnify this convergence points, we do not find any contact and the convergence tendency infinitely continues.

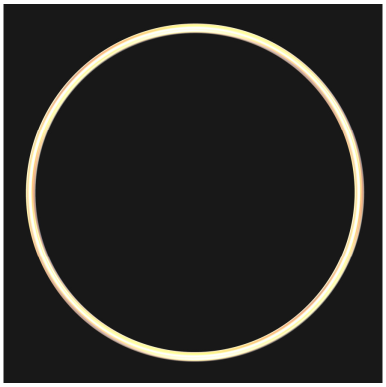

Figure 1: *A simple circle appearing as a ring is the initiator of the fractal.*

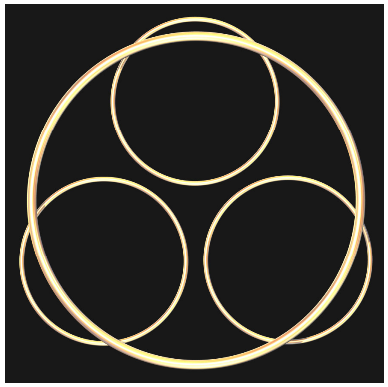

Figure 2: *Three smaller and similar circles with mild distances and diameters half of the diameter of the initiator circle play role as generator. They are placed under the initiator so that their smaller parts settle outside the perimeter of the initiator circle. (Iteration 1 with 4 circles (Exceptionally in addition to the initiator))*

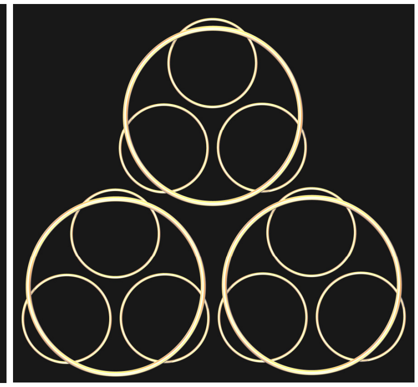

Figure 3: *If we put new generators of triadic circles under each of the circles provided in the previous iteration, the third iteration is provided. The initiator large circle has been also removed to have a better view of fractal aesthetically. (Iteration 2 with 12 circles)*

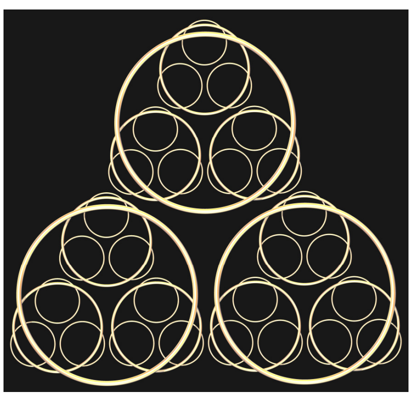

Figure 4: *(Iteration 3 with 39 circles)*

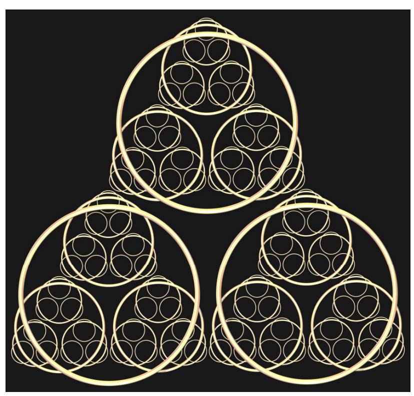

Figure 5: *(Iteration 4 with120 circles)*

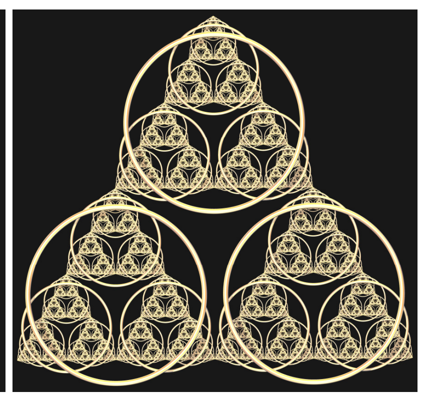

Figure 6*: By intermediation of smaller circles as further iterations, toothed large and small lines are changed to more straight ones, thus we have better representations of triangles in this level. (Iteration 10 with 88572 circles)*

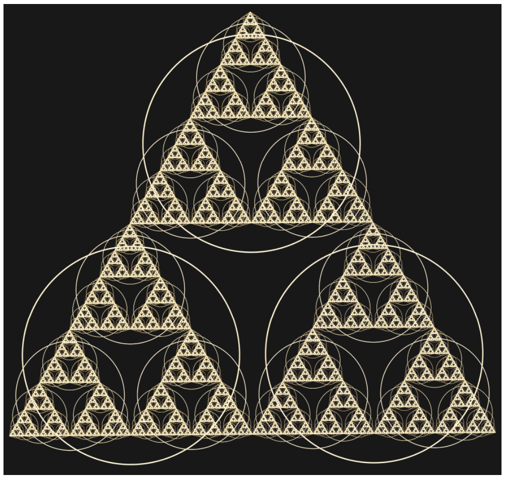

Figure 7: *Reducing the thickness of constructive rings and propelling them to circles with the least visible thickness on the screen, our Sierpinski fractal offers a better exposure. (Iteration 20 with 5230176600 circles)*

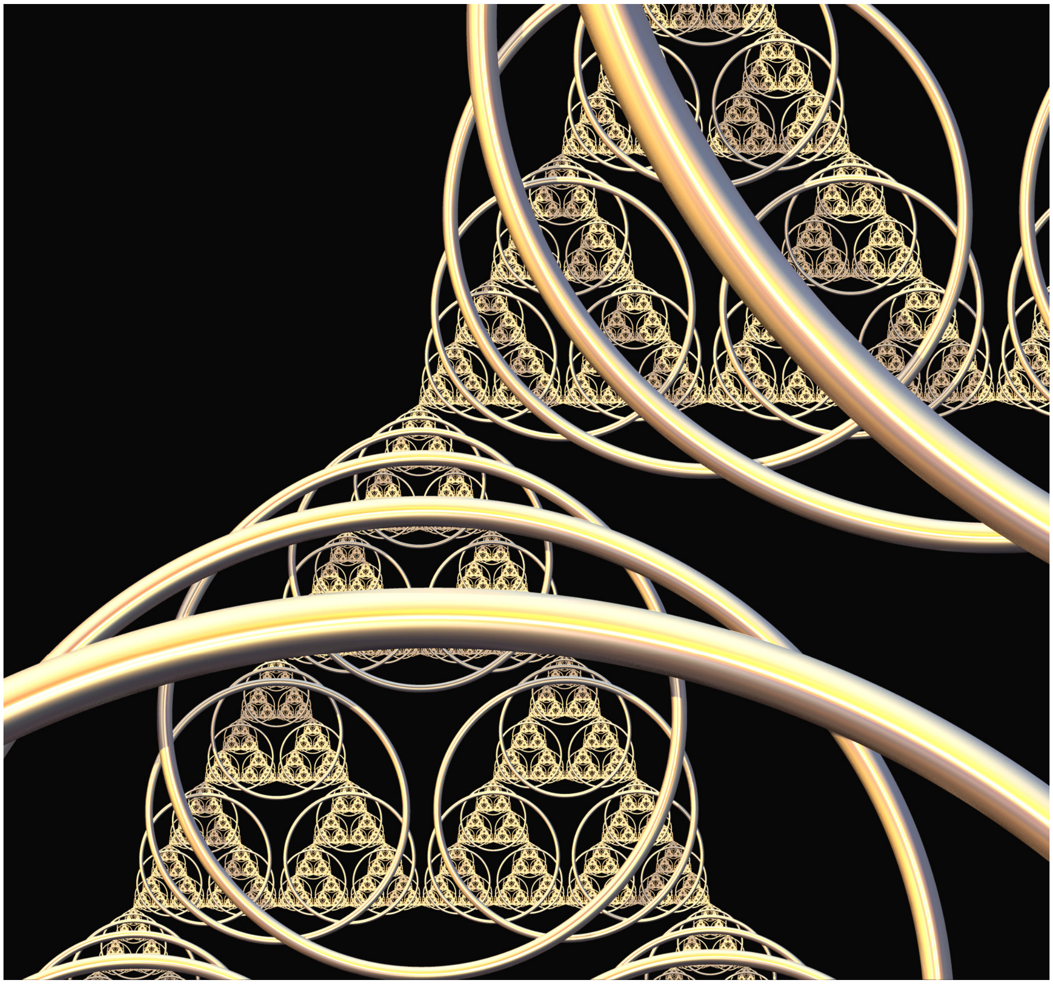

Figure 8: *A detail of Figure 7 with slight exaggeration on shadows and lightings is shown to have a better perception of three dimensionality of overlaying rings and the delusive generated vanishing point.*

Watching each vanishing point, like what is seen in Figure 8, apparently admits we are looking at a real 3D shape. However, it is just an illusive representation of a perspective depicted on a 2D plane. Even though, at first it could be concluded that the whole shape could be rebuilt as a 3D installation ensemble, composed of distinct layers of circles deputing different iterations, in real world by exploiting perspective rules like assuming the first layer of rings as an image plane and calculating other sizes and distances related to farther layers proportionally [3]. If we imagine all circles participating in any iterations of the construction of the fractal as distinct layers with specific distances in the space, we will have a better perception of real tunnels in a three dimensional reality. Our image contains infinite vanishing points and any single ring is a part of a tunnel. Hence, what we have is the infinity of tunnels with an infinity of vanishing points. In other words, we have infinite parallel tunnels with the infinity of vanishing points placed on infinite vanishing lines or eye levels. Even if only a part of the image is assumed as a temporary target of quest, that small proportion will contain infinite vanishing points and lines in it as well. But, according to perspective rules dominating our real world, all lines which in reality are parallel will converge toward a single vanishing point [4]. It is expectable that the key of the precise self-similarity and repetition appearing everywhere of the image is caused by breaking perspective rules governing our real world. Such a spectacular experience of having an infinity of vanishing points will be just possible in visualizing hyperbolic spaces in digital worlds forever. These are bizarre artistic aspects of virtual installations which challenge the strictest confines of the dimension we are living in.

Another strange matter lies in the way of the branching of tunnels in this fractal. Actually every tunnel inevitably branches to three similar subbranches while the mother tunnel, by keeping the distance and size proportions accurately, remains visually a part of each of child branches. In reality parallel branching causes completely different visual behaviors and for example three parallel tunnels can not have a common beginning ring. Continuing the iterations in the fractal, each child itself branches to smaller parallel children and so on. It is well known that although traveling toward the inside of the image increases the number of the subtunnels, none of them touch each other whatsoever. Therefore, looking inversely from another side of this tunnel network, supposing tunnels as simple straight lines, we will have an infinite number of parallel lines converging toward a singular point assumed as the most outside ring which is our first generator. In a 3D world, the only point at which any two or more lines that are in reality parallel, if extended indefinitely, will appear to converge is the vanishing point [4]. Consequently, we seem to be sitting at the vanishing point in a 3D world which has no meaning in our real world but maybe in contractive spherical hyperbolic spaces. Because, vanishing point is a virtual point at the other infinite side of observer's observation and never exists in reality.

Fractals, even those well known, still have several undisclosed aesthetical aspects which need investigating to introduce eventualities only possible in modern mathematics in cooperation with modern computers. The idea of this work was basically inspired by Felice Varini's delusive environmental installations and paintings that challenge the dimension notion in the eyes of the viewers [5].

References

[1] M. Garousi, H. D. Dezfoolian, *Constructing Sierpinski Triangle with Rings*. In G. W. Hart, R. Sarhangi, 2010 Bridges Proceedings, Tessellations Publishing, Arizona, 2010, pp. 411-414.

[2] http://en.wikipedia.org/wiki/Iterated_function_system .

[3] B. MacEvoy, *Central Perspective*, 2007. Available at http://www.handprint.com/HP/WCL/perspect2.html .

[4] J. D'Amelio, S. Hohauser, *Perspective Drawing Handbook*, Dover Publications, NY, p. 23, 2004.

[5] Felice Varini: http://www.varini.org/ .