Sundials from Toroid Surfaces

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Abstract

The circadian and annual geometry of the sun's rays will simplify into what is termed the shadow planes model for sundial projection. The author employs this model within a 3-D modeling program to design a series of non-traditional sundials derived from toroidal surfaces. These include the standard torus, as well as one eccentric torus: the horn cyclide.

Introduction

In order to mark the hour lines and day lines of sundials whose surfaces are curved – in this case toroidal – a simple abstracting of solar angles into the classic shadow planes model for sundial projection is indispensable. This model consists of a series of triangular planes that denote the set of all apparent solar angles at given moments each day of the year. The hourly version of this model, then, comprises 24 such planes in polar array with the apex of each triangle meeting at the center. As part of designing a dial in a in a 3D modeling environment this model is built with its center on the nodus, or point, of the intended dial's gnomon. The axis of the array points to the north at a rising angle equal to the latitude of the site.

Once positioned, the intersection of these planes with any surface of the designer's choice will yield the exact hour lines for the proposed dial. This frees the designer to mark exceptionally elegant surfaces while maintaining accuracy. The two sample designs below focus on toroidal surfaces. These are the standard "donut" torus and the horn cyclide.

Using the Shadow Planes

The earth tilts relative to the plane of its orbit around the sun at an angle of 23.45°. Called the ecliptic, this angle causes the sun's declination to increase from its median by 23.45° in summer and to decrease by 23.45° in winter (Figure 1). The declination of the sun then varies by 46.90° each year. For any location the mean declination is the co-latitude, i.e., the complement of the latitude of the site.
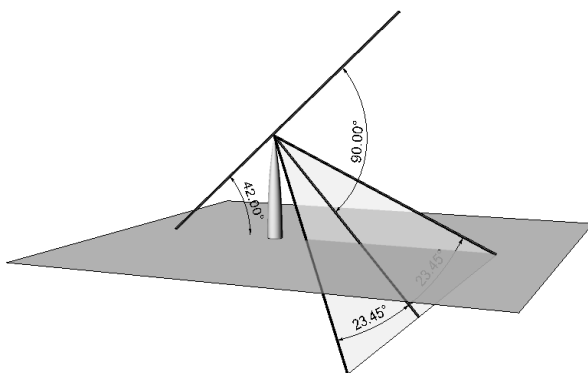

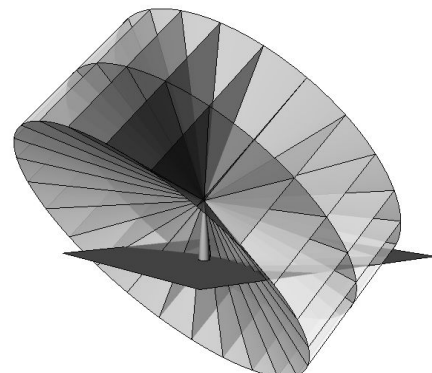**Figure 1. Noon shadow plane at 42° latitude.**

Figure 2. Hourly shadow plane array at 42° latitude.

Figure 2 above illustrates the full set of hourly shadow planes, all centered on the nodus and angled north at 42° declination. Note that the model depicted includes upper and lower conical surfaces and also a median disc. These surfaces were swept from the upper, lower and median lines used to construct the noon shadow plane. The surfaces describe the daily path of the sun's rays on the summer solstice, the winter solstice and the equinoxes. Once properly placed and aligned to intersect with the dial's surface, the Intersection command creates lines (Figure 3) where the model's surfaces and the dial face intersected. These then form the daily hour lines and the day lines for the solstices and the equinoxes.
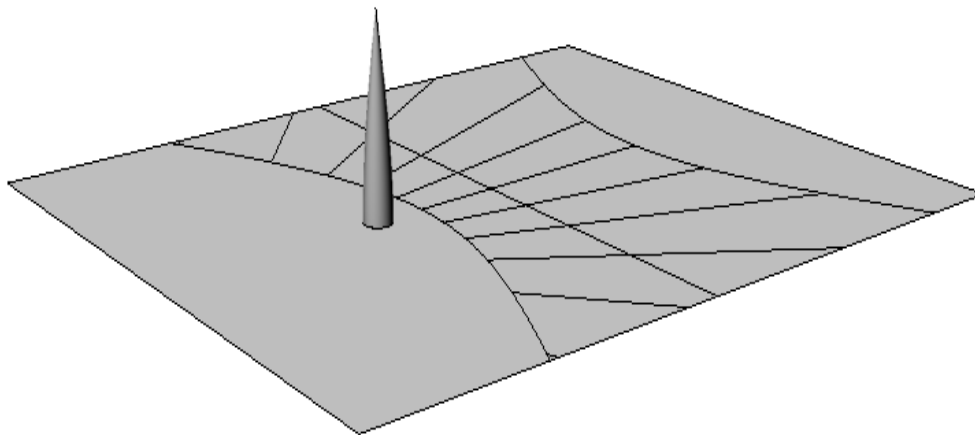

Figure 3. Intersection lines from Figure 2.

Torus Dial

A torus, whose inner radius is half of its outer radius, will, if sliced tangent to its inner perimeter (Figure 4), yield the lemniscate of Bernoulli Its the crossing point may become the nodus for a sundial projected onto the inner face of the torus. A goal of this design was to create a dial whose face and gnomon were continuous. Figure 5 below reveals the lemniscate; Figure 6 has the shadow plane model inserted at the nodus of the lemma; and Figure 7 holds the resulting hour and day lines created by the intersections in 6.\
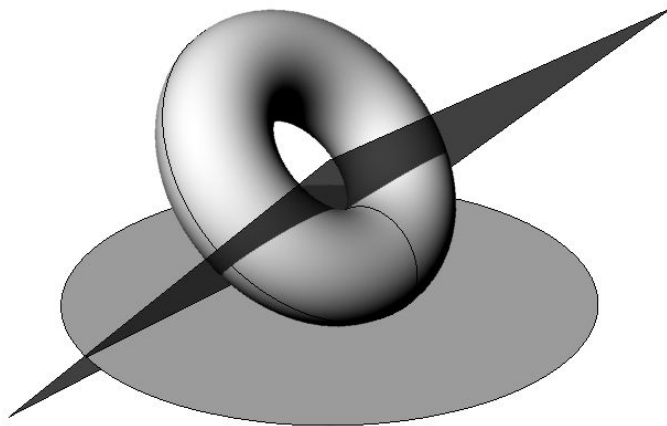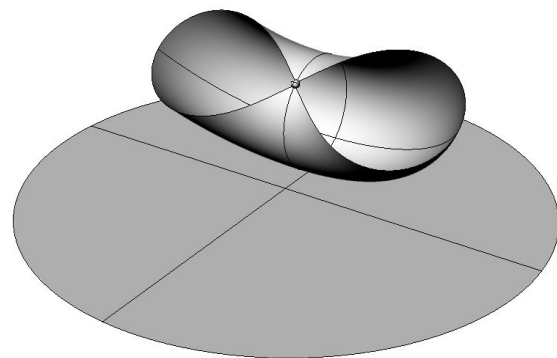

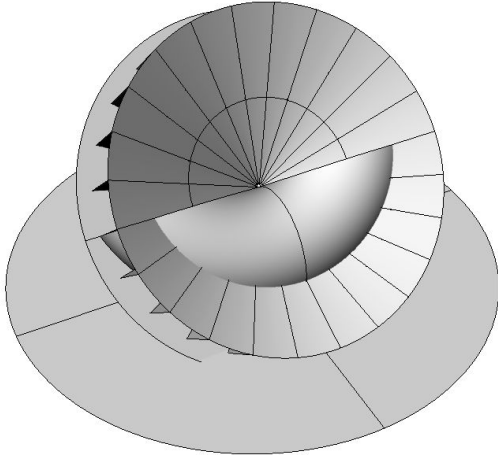**Figure 4. Torus sliced tangent to inner perimeter**   **Figure 5. Torus segment from Figure 4.**

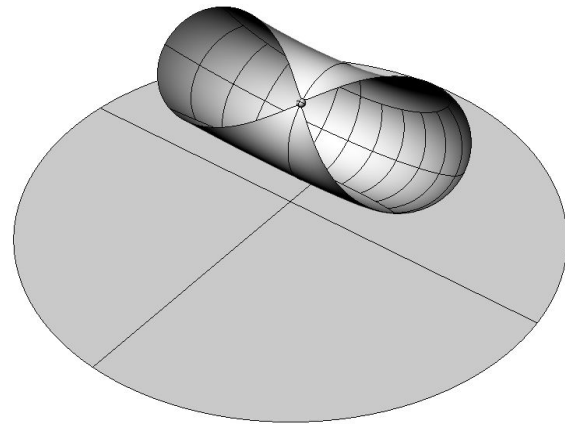**Figure 6. Torus segment with shadow planes.**   **Figure 7. Torus segment with dial markings.**

A form similar to the lemniscate, the analemma, is a standard feature of sundials that often replaces hour lines on a dial. The mean length of a day is 24 hours but over the year some days run fast while others run slow. This is a combined effect of the epicyclic motion of a point on the earth's surface as it revolves while orbiting the sun and the elliptical shape of that orbit. Plotting this differential over an annual cycle produces a distorted figure 8. Since the gnomon is itself lemniscate, it seemed appropriate that the theme be continued in the hour lines of the final design (Figure 8). In order to obtain these, lemniscate cones replaced the shadow planes in the model (Figure 9). The same Intersection command drew an analemma at each intersection with the torus.
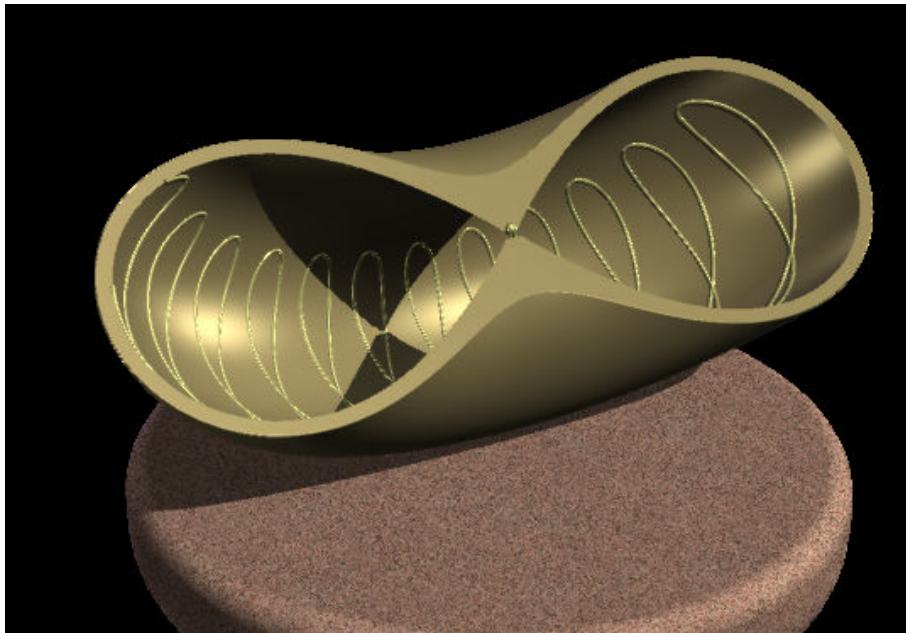

Figure 8. Rendering of torus dial in bronze.

Cyclide Dial

The cyclides are a category of quartic surfaces often referred to as eccentric tori, producing lopsided donuts fat on one end and skinny on the other. In the cyclide chosen for this dial, the horn cyclide, the

skinny end narrows to a point creating a croissant shape. The singularity at the meeting of the horns thus formed offers a built in nodus. Unlike the previous torus, the shadow planes (Figure 10) of this design project from the nodus to the outside surface of the cyclide.

The finished design for this dial (Figure 11) has the further advantage of serving as a universal dial. It may be rotated higher and lower as the latitude changes and rotated east to west as the longitude changes.
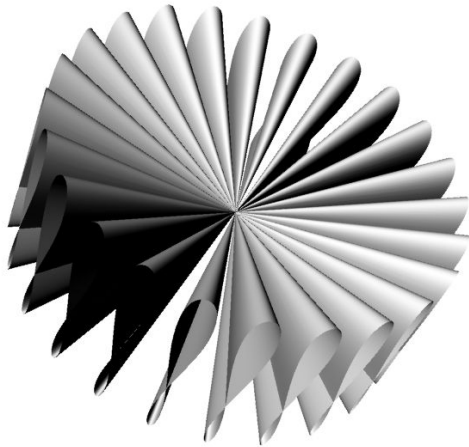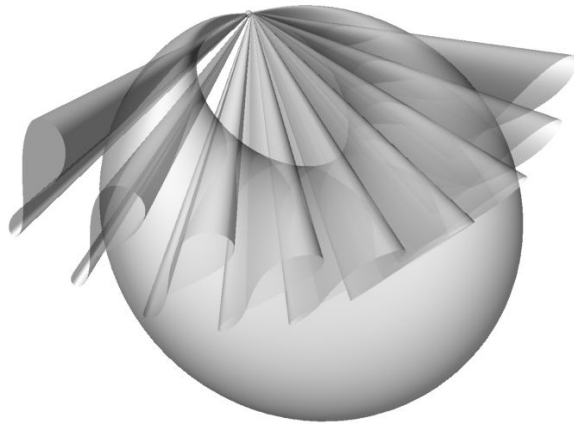

Figure 9. Analemmic shadow planes.

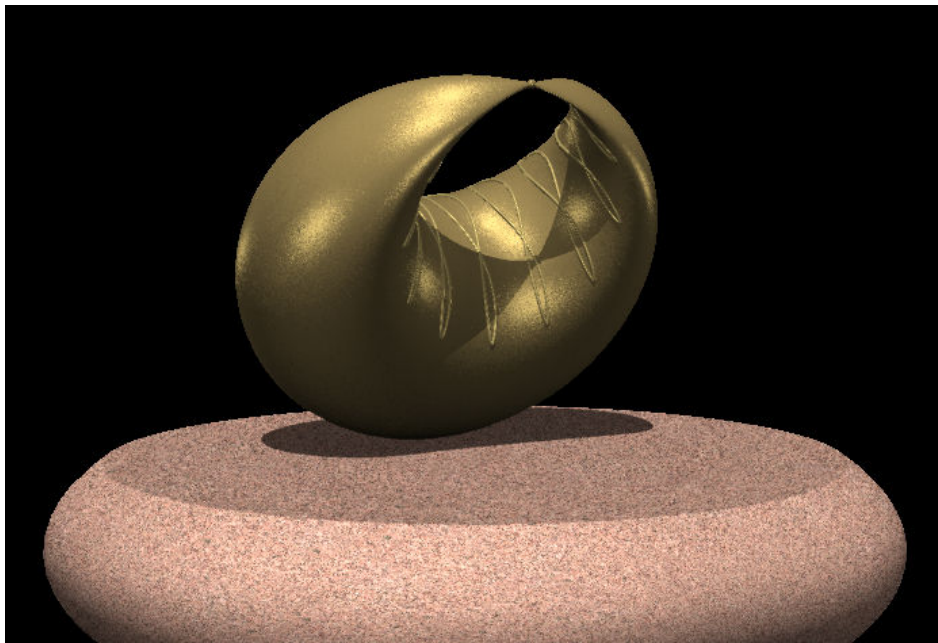**Figure 10. Cyclide with analemmic shadow planes**

Figure 11. Rendering of cyclide dial in bronze.   .

Conclusion

Sundials possess a grand eloquence simply stated. A handful of lines projected from the apparent conical path of the sun will act as a touchstone to reveal the inherent order and beauty of our universe. When that touchstone possesses the added beauty that inheres in the visualization of abstract mathematical surfaces, the eloquence, I believe, grows even grander.