Unusual Tilings and Transformations

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Abstract

I saw a spherical icosidodecahedron in 1956. Later I have had some insights, which have been new for me. The most common cognition has been, that any single answer brings more than one new question. In this paper I am listing some of both. Examples: – Do rhombic triacontahedra fill the 3D-space twice? – Concave rhombic triacontahedron as a single 3D-space filler. – A slim pentagon as a single aperiodic 2D-space filler with 109 different vertices. – The biggest cuboctahedron inside of an icosidodecahedron seems related with the slim pentagon. – Pull a dodecahedron to be a tetrahedron. – Move/turn the faces/edges of any Platonic, Archimedean or Kepler-Poinsot solid or their duals to create any other of those.

Stretch Dodecahedron to Tetrahedron

Build a dodecahedron by using edges only, no faces. Allow the angles between the edges vary. Select four vertices as far from each other as possible and pull them outwards.

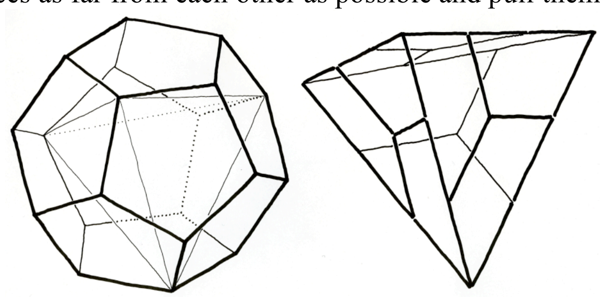

Figure 1: *A dodecahedron with interior tetrahedron and a tetrahedral transformation.*

A Twice-Filler?

The dual of the Archimedean solid icosidodecahedron is rhombic triacontahedron. Its dihedral angle is 144 °. 5 x 144 ° = 720 ° = 2 x 360 °.

Let us rotate a rhombic triacontahedron around one of its edges in five steps, each 144 °, and copy each step to have five triacontahedra with one common edge. If we continue with those five steps around all other edges of the original polyhedron and, moreover, around all edges of the ”step” polyhedra etc., does the operation ever end? Do the polyhedra cut each other into ”smallest units”, which cannot be cut anymore by the described operation? If yes, we may find a somewhat elementary set of space fillers. If not, we fill the space unlimited times with infinite number of edges of rhombic triacontahedra.

Rombic Triacontahedron as a Space Filler

Find four vertices of a rhombic triacontahedron, as far from each other as possible. Select only those surrounded by three faces. (The vertices are also vertices of an interior tetrahedron.)

Separate those four three-face elements. Turn them inside out and push them back in their original locations. The result is another rhombic polyhedron with four concave vertices. It is a periodic 3D-space filler. (Models available for testing.)

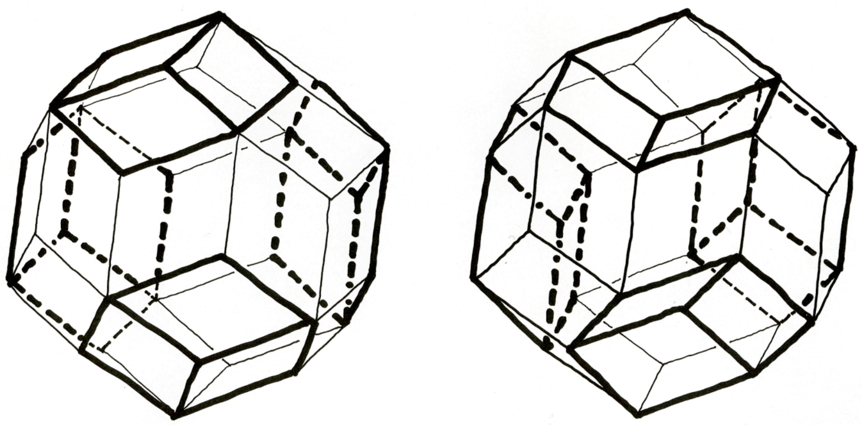

Figure 2: *A rhombic triacontahedron, four "tetrahedral" vertices drawn with thicker lines. Those trifacial vertices are then turned inside out in order to produce a 3D space filler.*

Slim Pentagon as a Space Filler

Turn one angle of a regular pentagon "inside". Two opposite angles remain 108 ° each, but two neighbouring angles will be 36 ° each only. The turned angle is 360 ° – 108 ° = 252 °. The angles are multiplications of 36 °, 1 x , 3 x and 7 x.

This "slim pentagon" can be tiled to fill the 2D-space aperiodically. The number of different vertices is 109, when rotations have been factored out, but mirror images not. Each vertex can be applied in aperiodic tilings. The kites and arrows have seven vertices.

(109 vertex figures with and without the surrounding tiles to be shown.)

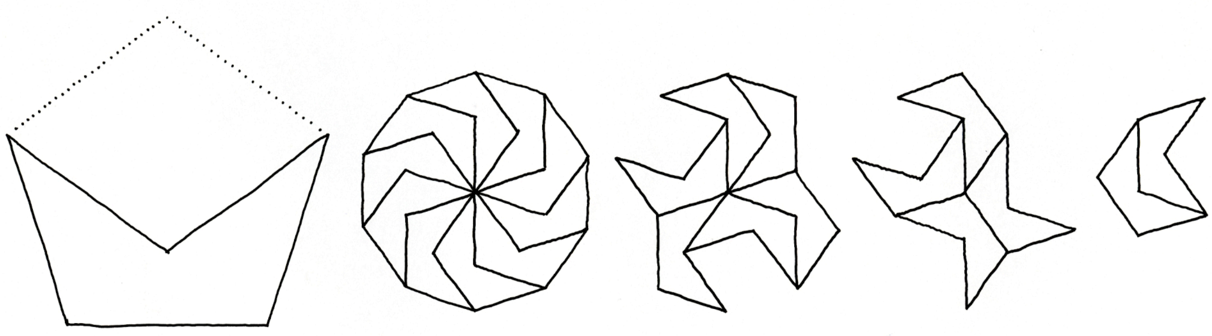

Figure 3: *A slim pentagon and four examples of vertices.*

Greatest Cuboctahedron Inside Icosidodecahedron

Consider the pentagonal faces of the icosidodecahedron as six pairs, each pair meeting in one vertex. Then replace the pentagons with slim pentagons in such a way, that the 252 ° angles are opposite to each other on both sides of the previously common vertex.

Finally use line segments to connect each 252 ° corner with all surrounding 252 ° corners. The line segments are edges of the maximum cuboctahedron inside of the icosidodecahedron.

(Model available for testing.)

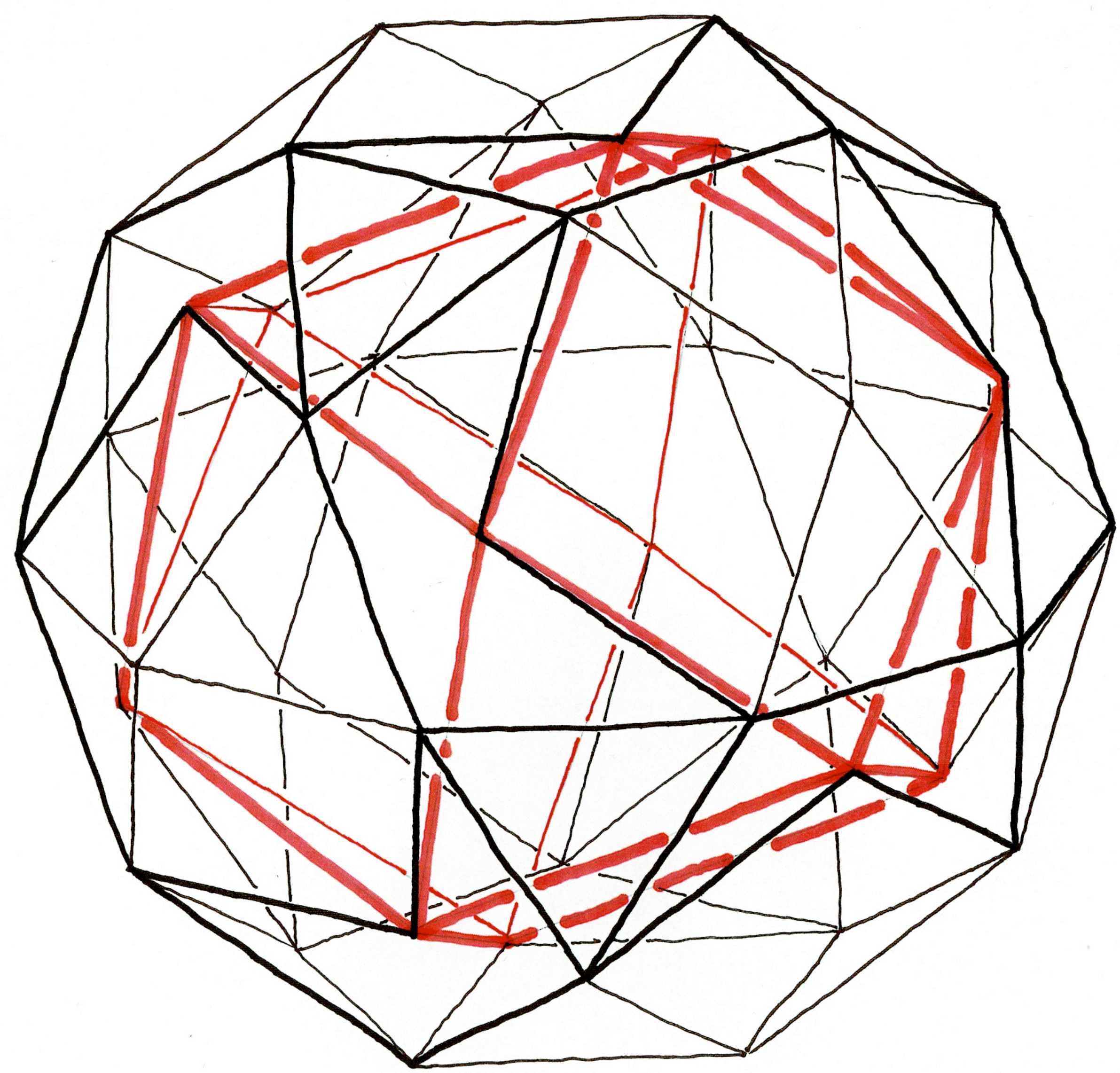

Figure 4: *An icosidodecahedron with slim pentagon faces, and an interior cuboctahedron, whose vertices coincide the 252 ° corners of the slim pentagon faces.*

Move and Turn

Any member of the group of the Platonic, stellated regular, and Archimedean solids and their duals can be produced from any other member of the mentioned group by applying one or both of the two operations *Move* and *Turn*.

Objects of the operations. The operations may be addressed either to the edges or to the faces of the solid, either to all of them or to a specific category of them. During the transformation, operations may be repeated and their objects changed.

Move. In this description, *move* means moving in or out, towards or away from the centre point of the solid. The axis of the movement goes through the midpoints of the solid and the object. During move, the selected objects keep their size, shape and direction, but others may change.

Turn. In this description, *turn* means screwing the selected objects around an axis, which goes through the midpoints of the solid and the object. During turn, the selected objects keep their size and shape, but the others may change.

(A video to be shown.)

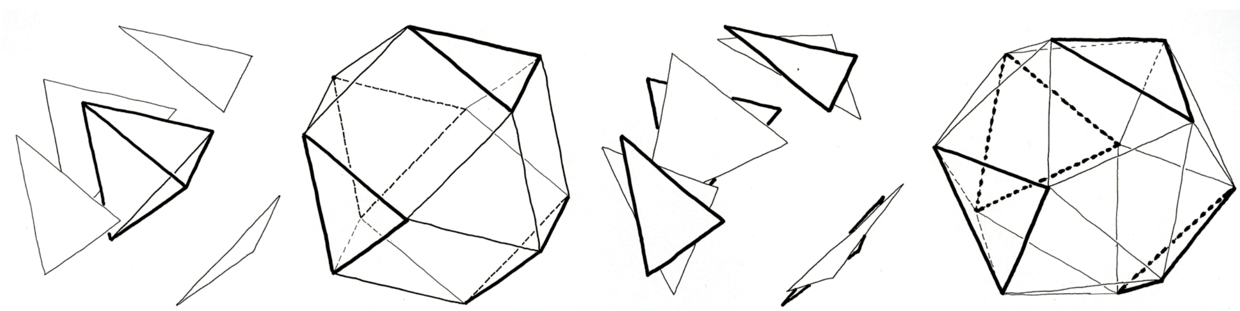

Figure 5:

*A tetrahedron and its faces **moved** outwards: six edges open to six square faces and four vertices open to four new triangle faces.*

The result is a cuboctahedron.

*Original tetrahedral faces **turned** anticlock-wise: each square face stretch and fold to two new triangle faces.*

The result is a snub tetrahedron or icosahedron.