Taking Sides, But Who's Counting? The Decagonal Tomb Tower at Maragha

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Abstract

Lu and Steinhardt introduced the term "girih tiles" to describe the set of equilateral polygons that structures a colorful two-dimensional decagonal tiling on the Darb-e Imam in Isfahan, Iran (1453 CE) with distant roots in the five-fold symmetries articulated in brick on the Gonbad-e Qabud, a tomb tower dated to the late 12th century CE located at Maragha in western Iran. Their work seeks to establish the early existence of quasi-crystalline tilings long before such means of covering the plane were understood mathematically in the West. Questions remained unanswered as to whether those who constructed these monuments were aware of the mathematical significance of their constructions. Lu and Steinhardt, as well as Makovicky and Bonner, who legitimately claim prior discovery of these decagonal tilings and their sub-grids, all missed the fact that the tower is itself decagonal. This brief paper draws attention to the relationships among architectural form, geometric ornamentation, and Qur'anic inscriptions in assessing the cultural significance of the Gonbad-e Qabud.

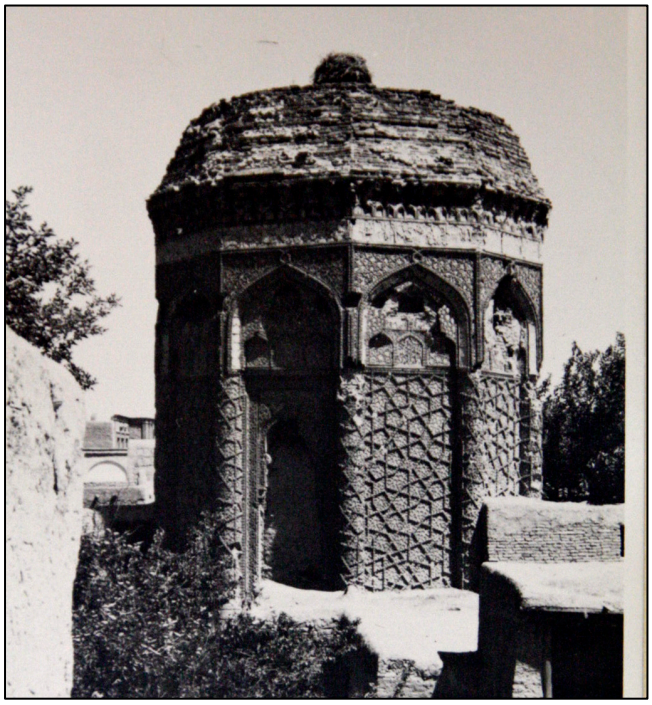
Preface

The Gonbad-e Qabud, a decagonal tomb tower at Maragha in western Iran that is dated to the late twelfth century, is the recent subject of heated debate concerning the periodicity of the five-fold symmetry of its baked brick ornament. What sparked the debate is an article [1], "Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture," by physicists Peter J. Lu and Paul J. Steinhardt, published in *Science* in 2007. Hot button topics were anticipated in "News of the Week: Mathematics," which appeared in the same issue [2]. A technical comment by Makovicky [3] was published several months later, accompanied by the authors' response [4]. Although Lu and Steinhardt dutifully cited [1:n18,n19] the earlier work of Makovicky from 1992 [5] and Bonner in 2003 [6], they had not credited either of these individuals with prior discovery of the tiling in Maragha first identified by Makovicky as quasi-periodic with five-fold symmetries, nor its historical development using what Bonner termed a sub-grid (corresponding to the "girih tiles" of [1]). Meanwhile, Lu and Steinhardt were soon widely cited [7,8,9,10] and the topics raised regarding quasi-periodicity quickly became classroom conversations and student projects around the world, in Egypt [11], U.A.E. [12], Singapore [13], and California [14].

Changing the Ground Rules

Throughout the debate, no one has noticed that the monument (fig.1) is itself *decagonal* in plan (fig.2). Prismatic in form, it has an engaged cylindrical buttress at each of ten exterior angles, and the roofing arrangement, as observed by M.B. Smith [15], gives evidence for a polyhedral pyramid of ten triangular facets (fig.1), originally covered with turquoise glazed bricks. The misconception that Gonbad-e Qabud is an octagonal building (normative for the period) is widespread [1,3,5,16], reiterating the mistakes of earlier publications [17,18,19,20]. Although Makovicky mentions visiting the site in 2002 [16:128], he

must not have stopped to count the sides! And Lu, who so carefully relied upon historical photographs for testing his reconstructions of the patterns [1,4], did not stop to think that the four sides as represented in these historical photographs could not signify an octagonal tower.



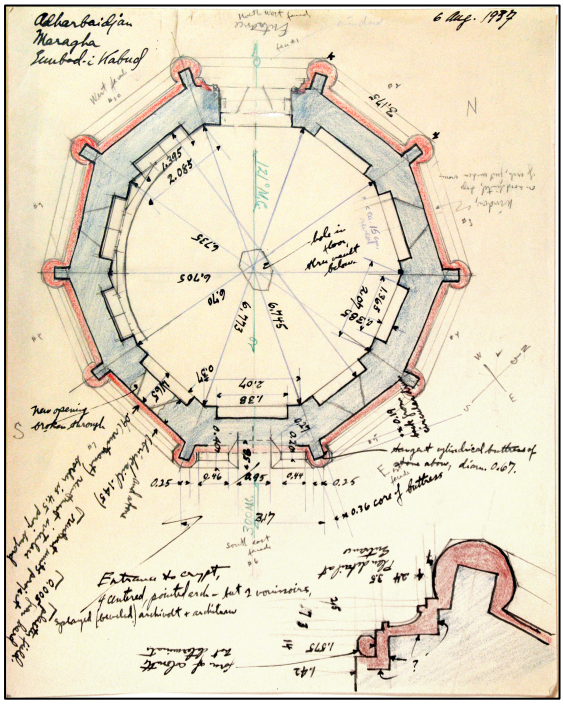


Figure 1 *(left): Maragha, Iran. Decagonal tomb tower, view from WNW. Photograph by M.B. Smith 4 Aug. 1937 [15].*
Figure 2 *(right): Maragha, Iran. Decagonal tomb tower. Annotated field plan by M.B. Smith, 6 Aug. 1937 [15].*

That the Gonbad-e Qabud (fig.1) is decagonal in plan was correctly recognized as early as 1937, recorded in the field drawings of M. B. Smith (fig.2), which are held in the archives of the Freer Gallery of Art and Arthur M. Sackler Gallery [15]. The collection includes a full set of Smith's black & white photographs of the monument and its decoration. In the 1980s Daneshvari [21] described the form as decagonal, presenting a more detailed sketch plan and elevation in [22], the correction to the literature concerning its decagonal plan made explicit in a review [23].

Modern sources for the history of Islamic art and architecture reveal confusion about the number of sides of this tower. Although ArchNet [24] designates the correct number of sides in its Digitial Library catalogue entry, it is listed as octagonal by Stronach and Young [17] and Blair [20]. Curiously, Hillenbrand [25] does not mention Gonbad-e Qabud, although it is included among the enumeration of Islamic monuments in Azerbaijan by Kleiss [26], without reference to its form. Hillenbrand does refer to Maragha as one of several towns listed in Anatolian inscriptions in which craftsmen are named (25: 370) and states that "most of the Anatolian mausolea of the pre-Ottoman period are tomb towers on the Iranian model," with an implication that the basic formula for this would be a cylindrical or polygonal body (usually octagonal) with a pyramidal or conical roof. The word "decagon" appears only once in Hillenbrand's index, and in the referenced page [25:157), it refers to a much later building with "an unusual ten-sided base."

With respect to the geometric ornamentation of the building, mathematicians have focused exclusively on the pentagonal/decagonal relationships in the broad band of ornament that wraps around the building in its lower reaches. Such statements regarding the repetition "around the eight-sided building," which Makovicky describes as an "architectural rather than a geometric problem" [3:1383a]

and reconstructions of the "seven decorated panels" [1:SOM, fig. S6] must now be corrected for a ten-sided building of which nine faces, rather than seven, bear this ornament, and including all ten engaged columns [15] (fig.1). Above this broad band of ornament there are other areas of exceptionally well-crafted designs with mathematical significance in their time and extraordinary for their complexity. Meriting particular notice are the aperiodic designs of the spandrels above the arches, which are reflected across the central vertical axis of each rectangular face. In each spandrel, i.e. filling the space between the arch and its rectangular enclosure, there is a sequence of five-, six-, and seven-pointed interlaced stars set within a polygonal net with a curvilinear overlay (recorded by Wilber [27:39,fig.17]) and mentioned by Makovicky [16:130]. Within the arched niche of each facet of the building there are three tiers of a shallow *muqarnas* that comprise a five-lobed arch; above these inner arches, there is a progression of eight-, nine-, and (half) ten-pointed interlaced stars with petal-sharing [18:pl.36]. These designs are repeated in each spandrel and above the inner arches, and seem to offer clever solutions to specific geometric problems that warrant further study. The intentional adjustments to the geometry of these progressive sequences, as well as the critical adjustments to the pattern with pentagonal and decagonal symmetries [1,5] surely indicate liberties taken with full awareness on the part of skilled craftsmen playing within the limits of plane geometry, even if these details may not be visible to the casual observer. Gonbad-e Qabud, indeed, deserves a prominent place within the long line of geometric experimentation and expression in the evolution of Seljuk and pre-Mongol ornament [28].

Prescience in Qur'anic Excerpts

There are several Qur'anic inscriptions on Gonbad-e Qabud [15,22,24,29]. That in the interior chamber [30:Ch.67v.1-4] may be interpreted as referring specifically to the complex architectural decoration on the exterior façade, with its several adjustments to the patterns, which could be considered flaws:

In the name of God, most gracious, most merciful!

1. Blessed be He in Whose hands Is Dominion; And He over all things hath Power;
2. He Who created Death and Life, that He may try which of you is best in deed: And he is the Exalted in Might, Oft-Forgiving.
3. He Who created the seven heavens one above another; No want of proportion wilt thou see in the Creation of (God) Most Gracious. So turn thy vision again: Seest thou any flaw?
4. Again turn thy vision a second time: [it] will come back to thee dull and discomfited, in a state worn out.

While architectural beauty, historical context, and commemorative significance are themes treated at length in the literature on the history of art and architecture, what is often lacking is recognition of the cultural significance of architectural monuments and their decoration in relation to the history of mathematics, to which this Qur'anic passage seems to allude. Parallel developments in art and mathematics in the eleventh and twelfth centuries have yet to be treated in an integrated manner. Within the development of pre-Mongol brick, glazed ceramic, and stucco decoration, visual solutions to geometric problems warrant further study [31,32], with a view towards understanding relationships among numbers, shapes, and the nature of space [8] and new advances in algebra and geometry [33,34]. Soon after the building of Gonbad-e Qabud, Maragha became the capital of Hulegu, Ilkhanid ruler of Iran in the thirteenth century, where he established an observatory and appointed Nasir al-Din al-Tusi as chief astronomer. Within the annals of the history of science, Maragha became a major center of exploration and discovery [19,31,35]. It may be the case that the decagonal Gonbad-e Qabud and its exceptional five-fold symmetries and playful geometric ornament reflect the achievements of a scientific community in Maragha that preceded the establishment of its famous astronomical observatory in the thirteenth century.

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