Using Triangle Parts to Create a Paper Quilt

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Abstract

The use of problem solving with geometric constructions can provide a rich source of fascinating and motivating problems for the classroom. The problems presented here form a novel way to investigate and encourage students to think creatively. With only three triangle parts given, the participants will find the triangle that fits the given information. Constructions can be placed in a variety of places in the geometry curriculum as they reinforce the theorems studied.

1. Compass Construction in the Geometry Classroom

A conference paper written by Reza Sarhangi for the Bridges Donastia 2007 conference [9] encourages the use of compass constructions in the geometry class for university level mathematics classes. I found that the use of constructions in the high school geometry curriculum added much interest and excitement to the study of geometry. Sarhangi states that "Geometric constructions have formed a substantial part of mathematics trainings of mathematicians throughout history. Nevertheless, today we are witnessing a lack of attention in colleges and universities to the importance of geometric constructions and geometry as whole, including the role of the axiomatic system in shaping our understanding of mathematics."
I found the same situation at the high school level. This paper seeks to encourage mathematics teachers of all levels to include geometric constructions in the development of the geometry they are teaching. The constructions are logical and orderly. Nothing is accepted because "it looks correct." Each step is to be documented and based on previously developed axioms, theorems or postulates. The tools for compass construction are portable. Students enjoy applying ideas to hands-on experiences with the compass and straightedge.

2. Getting Started

As we begin to work with the triangles, I ask the students to sketch a triangle with the given information. In this manner the student knows that the construction will be a possible one. At various levels of education one can challenge the student to test whether the construction will be a possible one. At the high school level, I only worked with possible constructions. This is a good time to discuss the triangle inequality theorem.

We now know that there is a unique triangle that does exist with the given information for a specific triangle. The next idea I use is the fact that the sum of the angles of a triangle is a straight line. At this point, my teaching book has not covered parallels, so we work without the idea of parallel lines. If the unit is taught using the parallel postulate, the problem solving is not as rich as it would be without parallels.

In some unusual cases, more than one triangle can be constructed. Encourage students to find a second triangle that fits the given information. Most of the problems only have one triangle solution. Reflections of triangles can be cited, but for our use we considered the triangles congruent.

As the student figures out where to start the construction, I have the student write in words and symbols their strategy. This is not a formal proof, but it does allow the student to organize their thought process in an orderly and logical way.

3. Notation

<A, <B, <C	angle A, angle B, angle C
a, b, c	side opposite angle A, side opposite angle B, side opposite angle C
h_a, h_b, h_c	altitude to side a, altitude to side b, altitude to side c
m_a, m_b, m_c	median to side a, median to side b, median to side c
t_a, t_b, t_c	length of the angle bisector of angle A, angle B, angle C

4. Sample Problems

Sample 1: Given <B, a, h_a. Construct a triangle with the given information

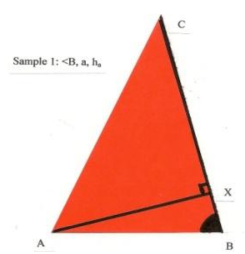


Figure 1: *Sketch of the given information for Sample 1*

As the student begins to work on a solution to the given information the sketch is essential. Most students begin the "construction" at an incorrect place. When using the sketching technique, the student will generally catch the error that they have made. As the construction "puzzle" is analyzed, the steps of the construction are documented in an orderly and accurate order.

1. Construct line *l*. At point X construct a perpendicular facing left.
2. Using point X as a starting point, mark off the length of AX. Point A is now determined.
3. Find <XAB by constructing the complement of <B.
4. Duplicate the complement of <B at point A using AX as one side to the angle.
5. Extend the construction in #4 so that line *l* is met by the extension of <XAB. Label point B.
6. On line *l* using point B as a start, mark off BC upwards to determine point C.
7. Triangle ABC is the required triangle

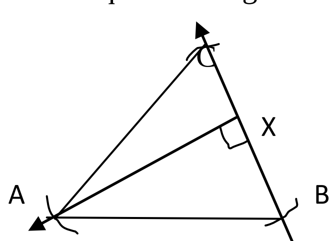


Figure 2: *Actual Compass Construction Listing the Steps*

Paper piecing the triangle of felt or fabric can now be done on a paper or fabric squares.

Sample 2: Given <B, a, m_a. Construct a triangle with the given information.

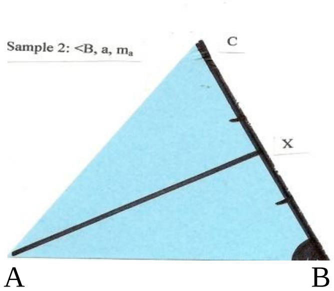


Figure 3: *Sketch of given information for Sample 2*

1. The triangle construction may be started in a number of places. We will begin with constructing side BC. The midpoint of BC or X can now be found. X should be labeled.
2. At point X, swing the length of m_a. Only an arc will show at this point.
3. From point B, duplicated <B facing down. Extend the side of <CBA until the extension meets the arc from step #2.
4. Now A is determined from the intersection of ray BA and the arc from point X.
5. Connect A to C and now triangle ABC is the required triangle.

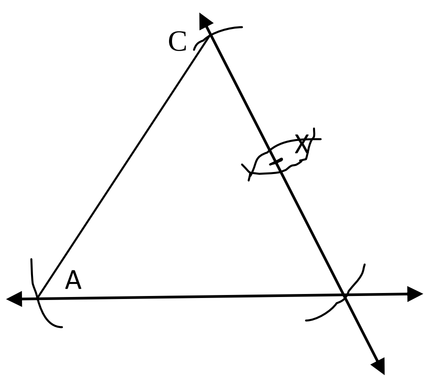


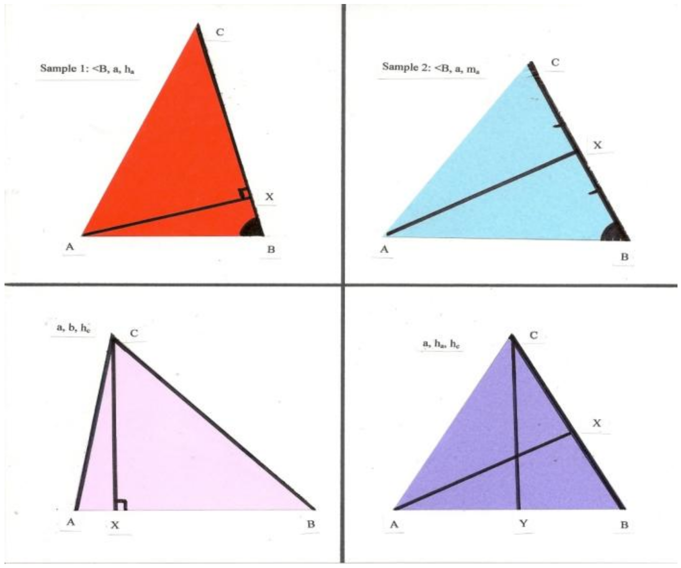
Figure 4: *Actual Compass Construction Listing the Steps*

Again, the triangle may be paper pieced or fused onto paper or fabric.

5. Other Sample Problems for the Investigation

<B, a, t_c	a, b, h_c	<A, h_a, b	a, m_a, h_a	a+b, c, <A
a, b, m	a, m_a, c	<B, a, h_b	b, t_a, h_a	<A, <B, h_a
a, b, h_a	a, b, m_b	b, <C, t_c	a, h_a, h_c	<A, <B, t_a
a, <A, h_c	a, b, h_b	<A, <B, t_c	a, t_b, h_c	<C, t_c, h_c

For more constructions see Posamentier and Wernick [3], Berzsenyi [6], Meyers and Wernick [7], and Wernick [8]. The paper triangles, along with their construction steps, can be assembled on colored, white, or patterned paper or fabric. Together the panels can be assembled to form a classroom "quilt". Here is a sample of 4 construction problems made into a paper quilt.



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