Amateur and Pioneer: Simon Stevin (ca. 1548–1620) about Music Theory

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Abstract

Simon Stevin (1558/59–1620) was a mathematician, physicist and engineer with an extremely wide spectrum of interest. His music treatise titled *Vande spiegeling der Singconst* was written in ca. 1600 but remained unpublished until the end of the19th century. This was the first European writing which defined the equal temperament with mathematical exactness, involving the ratio $\sqrt[12]{2}$ for half-tones and chromatic steps. This paper tries to show parallels between his music theory and his general way of thinking. It describes the highly contradictory and changing practical requirements and demands a theory of tone systems and temperaments had to meet; and it explains how Stevin, sacrificing much of the practical needs of the musician of his day for the simplicity of theoretical construction, discovered ingeniously equal temperament which was justified and generally accepted only two centuries later, with the development of musical style.

Life and Work

Simon Stevin (Bruges, 1548 or 1549–The Hague or Leiden, 1620) was one of the most versatile brains of his day. He wrote 11 books on various topics as book-keeping, interest tables, mathematics (including trigonometry, geometry, number theory, arithmetics and algebra), mechanics, astronomy, architecture, geography, navigation, military technique and politics.

His major contribution to mathematical thinking was that he accepted all real numbers, including irrational and negative numbers as equivalent, as numbers “of equal rigts”. This philosophy must have been connected with his re-invention of the decimals, introduced much earlier by Chinese and Arab mathematicians but in Europe propagated widely and rapidly by his booklet *De Thiende*, (‘The tenth’). Among his other major accomplishments there is a method for finding approximate solutions to algebraic equations of all degrees; he experimented with dropping lead weights of various size to see if they fall at the same speed before Galilei; his theorem of the triangle of forces gave an impetus to statics; he formulated the principle of virtual translations correctly; and he recognized the hydrostatic paradox.

The Birth of Equal Temperament

It would have been surprising if a man of such a wide interest and originality of thinking had not been trying to meet the challenge of finding new correspondences between musical phenomena and quantities, a task which excited many theorists since the antiquity. He actually did it; and with a fascinating result. But he never published, or indeed probably never finished his manuscript about music theory entitled *Vande spiegheling der singconst* (approximately: 'on the theory of music'). Nevertheless, his views did not go unnoticed. We know a letter, sent to Stevin by a Dutch organist who denies or refines some views of his; whereas his younger contemporary Isaac Beeckman, and theorists like Mersenne and Descartes rejected his views.

It is clear from his texts is that he had read some of the works of the most important music theorists of the 16th century like Glareanus, Zarlino, and Vincenzo Galilei. His writing comprises the main text titled "On the Theory of Music", which, however, only discusses the question of pitches, scales and ratios in various approaches, skipping e. g. musical notation, the aspect of note values and time, counterpoint etc. etc.; and an apparently fragmentary "Appendix" whose scope should have been considerably wider, according to its planned content. The main text begins, as it seems to be appropriate to a mathematician, with a series of definitions. However, he does not start from the discussion of various musical intervals as it was customary then but from the concept of the major scale, based on the concept of the *step* ("trap" in Dutch), existing in a minor and a major variety ("cleentrap" and "groote trap").

1st Definition

> Step is the next subsequent ascent which one rises in natural singing, of which the smaller variety is called minor step, the larger, major step.

2nd Definition

> Natural singing is that which by an orderly ascent takes place as follows: two major steps, one minor, three major steps, one minor, two major steps, one minor, three major steps, one minor, and soon gradually, in orderly sequence.

Octave is the only interval other than a second which he characterizes in itself and not only as a relation ("verlycking") made up of steps or whole and half tones. It is actually defined as "seven steps" (or six whole notes) in the first place; but he remarks that the upper note is "very similar" to the lower one, and he admits that it corresponds to the 2:1 ratio of string lengths. (Stevin never talks about vibrations and their frequency but he regards the length of string connected to any given pitch, reciprocal with the frequency, a basic attribute of the given note.) This is not quite logical as he rejects any other suggestions for simple proportions in the case of other intervals. Those are simply put together from steps or whole and half notes: there exist no "empirical" fifths or thirds, just "speculative" ones. However, one chain is definitely missing: we get no explanation whatsoever, why the "steps" are just as big as they are. This is the natural way ("natuerlicke sang") and that's it. Moreover, there exists no distinction between a minor second and a chromatic step (e. g. C to C sharp) whose musical functions are entirely different.

Stevin's strategy is that of eliminating any complication from the way of his (second) postulate declaring that all whole tones respectively all semitones are equal to each other. In fact, it doesn't follow from *this* that whole tones are exactly twice as big as half-tones are but he actually means that, because in the subsequent explanation he writes down the magic number of $\sqrt[12]{2}$, or rather its reciprocal as he is calculating with string lengths rather than frequencies. He also enlists his speculative frequency proportions up tone octave as the various powers of $\sqrt[12]{2}$. The complicated system of practically existing intervals becomes as simple and homogeneous as the now privilegeless "society" of real numbers. *Equal temperament has been born.*

Contradictions

There are essential contradictions built into the Western tone systems, let them be pentatonic, diatonic or chromatic ones. But in order to understand these, first we have to put Stevin's approach from head to feet. In reality, it is not the fifth which is derived from the second (i. e. is put together from major and minor ones) but quite the other way round: the size of the seconds is derived from the fifth. In fact, the size and order of the steps in the above-mentioned scales is decided so that the highest possible number of fifths and fourths occurs between the notes of the scale (with very good approximation). Fifths and fourths (characterized by the proportions of 3:2 respectively 4:3) are physically and physiologically distinguished by the second and third simplest joint structure of harmonics (after the octave), due to the many coincidences in their systems of harmonics. These intervals–especially the fifth, of which the fourth is the inversion, complementing it to an octave–sound very clearly and radiantly and, being so characteristic, they are relatively easy to sing in tune both synchronically and melodically. While it would be of course hopeless to prove historically, with mathematical certainty, that fifths are "primary" and seconds are derivable from them, it is an especially convincing fact that all three basic scales mentioned before, the pentatonic ("the black keys on the piano"), the diatonic ("the white keys on the piano") and the chromatic (all keys of the piano) can be originated from a series of four, six and eleven *consecutive fifths* (i. e. five, seven and twelve consecutive notes) projected, via octave transpositions, into the same octave (e. g. into the "middle octave", from C^1 to C^2). Or to put it quite correctly: their basic shapes can be originated in this way, the values of the seconds (and of the fifths) in practice never differ too much from the values obtained in this way. But there are serious problems and contradictions in the details, and, as it normally happens, there exists no perfect compromise.

First of all, fifths and octaves, in the strict sense, are incommensurable. The unpleasant fact that twelve consecutive fifths make *somewhat* more than seven octaves, was already known to Pythagoras; the difference, slightly less than one quarter of a minor second, is called a Pythagorean comma. This doesn't cause much of a problem until the music you play doesn't employ more than eight or nine notes of the twelve-tone set (e. g. the "white keys", B flat and F sharp), which was the case until ca. the 14th century. Or until you build a keyboard instrument with twelve keys. Because if you do, and therefore you want a closed sequence of fifths instead of an infinite one, you are forced to manipulate with the exact size of the fifths.

But there is a contradiction worse than the one between the octave and the fifth. However, this one only manifested itself after harmony based on triads became a decisive element in European music. This problem did not exist at all until ca. 1000 a. D. while the music of the Mediterranean was basically monophonic; and during the first centuries of European polyphony when the third played a subordinate role and was treated rather as a dissonance than a consonance. But by Stevin's time the sweetness of the major third and the major triad saturated European polyphony (both sacred and secular), a tendency which owes a lot to England and which was called on the continent "contenance anglois".

The major third, similarly to the fifth, is strongly audible among the harmonics of any musical sound. (The harmonics are formed together with the basic note on any stringed ore wind instruments; their frequencies being the multiples of the basic tone.) Therefore it is an essential requirement in polyphonic music to play or sing "true" major thirds (and fifths). The reason is simple: if f is the frequency of a bass note and there is a major third of it, say two octaves higher in the violin or oboe or in the right hand of the keyboard instrument, then it will be clashing with the existing 5th overtone, having the frequency $5f$, of the bass note, unless the third is an exact major third of the ratio 5:4. This clashing causes a beat and the whole cloud of overtones of the chord becomes messy and loses its periodicity. (Because of interferences actually several beats of different speed will be generated.) The sound loses its shine and becomes blurred. *What has to be made unmistakably clear is that this is an objective, practical physical and physiological fact, having nothing to do with any speculative play with simple integers.*

Meeting the Challenge: the Art of Compromise

Now the main problem of tuning and temperament during the centuries was that if you generate a diatonic or chromatic scale, as described above, from consecutive fifths, then either using true fifths (3:2; "Pythagorean tuning") or slightly "narrower" fifths, diminished by 1/12 of the Pythagorean comma, thus making it possible that the circle closes and we get a twelve-tone scale ("equal temperament", suggested by Stevin and used generally for tuning pianos today), the major thirds will be way too wide, and audibly out of tune. The difference between the Pythagorean third (four consecutive fifths minus two octaves, 81/64) and the true major third (3/2 = 80/64) is a tiny interval with the ratio 81/80 and is called the syntonic comma. This is somewhat smaller than the Pythagorean comma, just a little bit wider than one fifth of the half-tone. In the case of equal temperament, the problem is almost as serious. As the fifths in this temperament are by ca. one-fiftieth of the half-tone narrower that true fifths, the major thirds in equal temperament (i. e. also on today's pianos) are by ca. 1/7 of the half-tone too wide.

While this problem has been recognized in Antique times, it was, as pointed out before, rather a speculative theoretical problem those times. However, a millennium and a half later this became the central and most chronic problem of intonation, as far as instruments with fixed tuning are concerned. For singers and for players of all unfretted stringed and plucked instruments (those instruments, like the violin, where the player can put his fingers on the fingerboard anywhere he likes) and of wind instruments, where the regulation of the "fine tuning" is possible, this was a non-issue as they are always re to follow their ears and produce any thirds or fifths or octaves in tune.

But the problem was unavoidable on keyboard instruments (mostly organs, clavichords and harpsichords those days) where you had to fix the pitch of the note belonging to a key for the whole piece or a concert or, in the case of organs, for much longer periods. The major thirds caused by true fifths were so wide that they were out of question; Pythagorean tuning, if it had existed in practice before, played no role in our period. The major thirds of equal temperament are almost as bad, and obviously that's why this option has never been mentioned before Simon Stevin, although it's hard to imagine that they never thought of this possibility.

The only thing they could do was to manipulate with fifths of different size. This was made possible by the fact that not all 12 notes were equally important or were used with the same frequency. In ancient monophony it was common that music pieces only used a scale derived from 7 or 8 notes ordered by consecutive fifths. (For 8 notes e. g.: C, D, E, F, G. A, B flat and B natural was typical). Such reduced tone sets were used in early polyphony in instrumental music, too (apart from early chromatic pieces but they were a curiosity and mostly occurred in vocal music). The tone sets used in particular pieces only slowly spread over the entire 12 notes, and not before the mid-Baroque era, i. e. after Stevin's time. What could be done was to use narrower fifths (and thus more acceptable thirds) among the commonly used notes and push the excess to the fifths between the unused notes, making them so wide that they were practically useless. Sometimes they were squeezing most of the residue into just one fifth, the “remotest” one, which was unbearably out of tune and they called it a “wolf fifth” as it was howling like a wolf.

A popular method of the 15th and 16th centuries, which led to an excellent result within the limited set of notes of the age was the meantone temperament. Most fifths were reduced by ca. 1/18 of the half-tone; this is still a quite acceptable fifth, whereas four consecutive ones make a perfect major third (plus two octaves). If you tune seven or eight consecutive fifths in this way, e. g. B flat–F–C–G–D–A–E–B–F sharp, then all the “important” thirds are in tune and you can distribute the excess among the four most “obscure” fifths.

As you arrive at more and more complicated temperaments, you will need more and more flexibility and creativity to accommodate to the requirements of particular keys or pieces. Even in the case of meantone temperament, you can decide to “shift” the series of the “good” fifths to the flat or to the sharp direction, deciding e. g. if a certain key will be can be used as a perfect E flat or a perfect D sharp. We can’t go into further details; but it may have become clear already that the more notes you are going to use frequently the more concessions you have to make as to the number and perfection of “good” thirds. And the music history was advancing exactly towards the “emancipation” of notes and remote keys.

Courage or Ignorance?

The idea of equal temperament (which was known to Chinese music theorists in the 16th century) must not have been a complete novelty for Stevin, too. It had probably been used “tacitly” on fretted plucked instruments like the lute well before him. You can also blame him with *declaring* the superiority of equal temperament against the exponents of other views instead of than using real arguments; e. g. he simply postulates that, in the case of the fifth, 3:2 is and *arbitrary approximation* of the true value, $\sqrt[12]{1/128}$.

It has to be pointed out that Stevin’s resentment against simple ratios is hypocritic as it reappears on a different level. The Pythagoreans thought (basically correctly) that the ratios of the two string lengths (to us, rather those of the frequencies) belonging to the two notes of an interval are simple fractions. However, with Stevin’s equal half-tones, with the whole-tones being exactly twice as much, the ratios of the permitted intervals–i. e. the ratios of the logarithms of frequency ratios–are simple fractions; e. g. a minor third is 5/3 of the fourth, or a fifth is 7/12 of the octave. These two kinds of simplicity are, unfortunately, incompatible. Stevin may have been the first one who understood that. But he absolutely does not deal with actual sounds, harmonics or beats; and he tries to play down the importance of the Pythagorean contradiction between octave and fifth, and conceals the phenomenon of the wolf fifths (in the case of using true fifths elsewhere) by simply alleging that the last fifth, the residuum is accidental because we cannot tune the other eleven fifth so exactly. (Of course, the inventors and users of many ingenious temperaments were even able to distribute the comma as they liked between the fifths, using the method of counting the beats which is an extremely precise method of measuring tiny pitch differences.)

While he is at least refusing the validity of the simple proportion in the case of the fifth, there is deep silence about the much more essential contradiction between the requirement of perfect thirds and equally tolerable fifths. He does not even mention thirds or the existence of meantone temperament.

But still, I think he was an original thinker and he conjectured much of the future. There unquestionably existed a Gordian knot: the difficulties of tuning various instruments were constantly growing with the gradual conquest of more and more keys and harmonies during music history. He seems to have recognized the importance of the problem. It is true: he did not bother with undoing the knot, he simply cut it in two. What is most interesting is that his suggestion did not match the musical style of his day at all. Disregarding the special case of the fretted instruments, the sacrifying of the perfection of the major thirds was still too high a price around 1600 to pay for pretty little advantages. However, values were changing. Bach's *well*-tempered clavier was not an *equally* tempered instrument yet. But it was already playable in all keys, even if not in an equally pleasant way. And another century later, for Schubert or Chopin, the possibility to play equally well in any key and, and, above all, the freedom to unlimited modulation, was already an absolute must. By then, the loss of the perfection of the major thirds was already a very reasonable price. The more so, as the overtones of the modern piano are slightly irregular from the beginning, this phenomenon tending to conceal minor defects of intonation.

That all happened two centuries after the death of Simon Stevin. As we see, he somehow succeeded in foreseeing the needs and requirements of the far future. This is which makes it interesting to us to conjure up his figure from the distance of four centuries.

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