Mathematical Balloon Twisting for Education

Vi Hart
http://vihart.com
E-mail: vi@vihart.com

Abstract

This paper is a reference for a hands-on workshop where participants will make polyhedra out of balloons, and learn how to use balloons to demonstrate mathematical principals, from preschool (counting, shape recognition) to college level (graph theory, theoretical computer science). Balloons are a great medium for teaching hands-on mathematics, whether to young students, who are more open to the idea that an octahedron (for example) is a beautiful object if it is made out of a balloon, to adults, who can find an interesting challenge in constructing complicated models. Balloons are also inexpensive, easy to obtain, and easy to work with (once you learn the basics, which will be covered in the workshop), which combined with their hands-on nature and visual appeal make them a great medium for teaching and learning mathematics.

Introduction

Balloon twisting has taken off relatively recently as an art form, and their use in polyhedral models is more recent still. Some instructions and examples of polyhedral models exist [1–5], and balloon models have been used for teaching molecular structures [6], but little has been written on how to use them to teach mathematics. This paper serves as a reference for skills learned in the workshop, including teaching young children to think geometrically as you make them balloon models, classroom activities that help students to understand graphs and explore some basic graph theory, and some examples of more difficult models and questions which provide a challenge even for experienced mathematicians.

The Balloon Octahedron is the Balloon Dog of Polyhedra

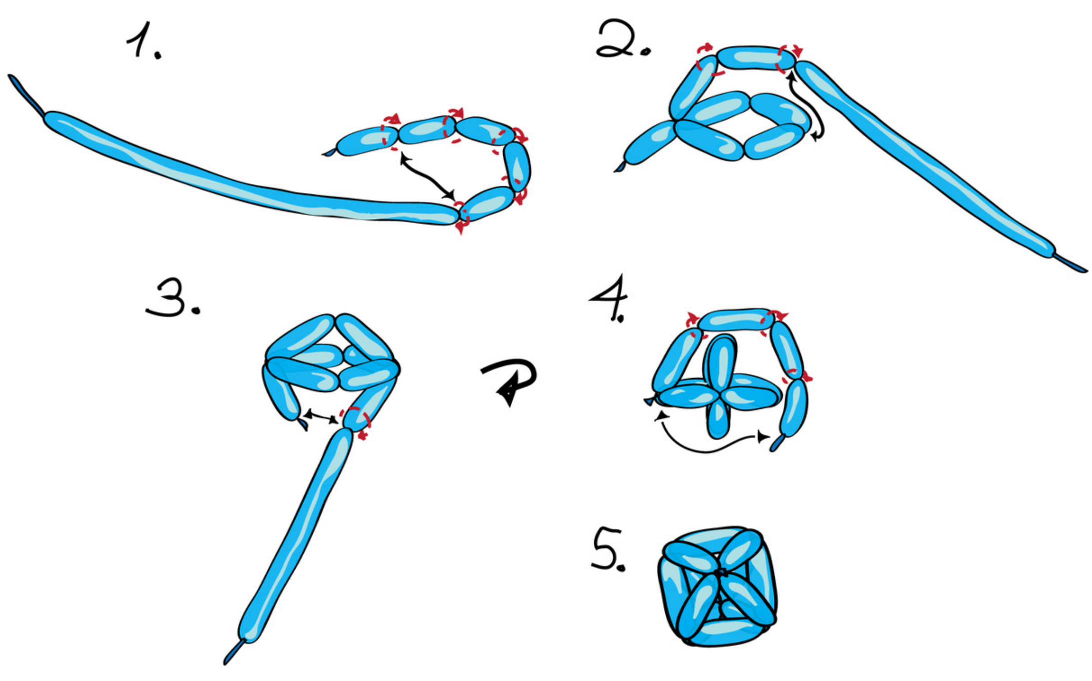

Figure 1: *How to make an octahedron out of one balloon.*

The octahedron is simple and, with practice, very fast to make. It is a great first model to learn, and once you learn the basics with the octahedron, it is easy to make more complicated models.

Practical tips. There are two standard sizes of long balloons, 160s ($1'' \times 60''$) and 260s ($2'' \times 60''$), both of which work to make the octahedron, though I prefer the skinnier 160s for this model. The more twists you make in a balloon, the more deflated tail you need to leave at the end for the air to expand into. With experience, one gets a feel for how much to leave for any given model. A section of balloon won't stay twisted unless it is twisted to another section or bent at an angle, so during step one in the above instructions, bunch up the sections back and forth in your hand as you twist them off, until you're ready to complete the square. I recommend using normal solid color balloons, not metallic or pearl colors, because metallic colors often aren't as stretchy and pop loudly enough to cause ear damage.

The magic balloon octahedron wand. The biggest difficulty is getting the edge lengths to be equal, which takes practice! This can be circumvented by twisting off small sections and then leaving a tail, which can act as the handle of a magic balloon octahedron wand (this requires 160s). This model is fast to make, so it is perfect for handing out at math and science-related events.

Knowledge is power. At any event with young children, they will automatically line up to have a magic octahedron wand made for them as soon as they see that you're giving them out. Once you have a captive audience, take advantage and teach them something about the octahedron as they watch you make it!

Figure 2: *Making a balloon octahedron wand, watched by a girl who just received one.*

If a child is very young, have them count the edges with you as you twist them off. Once they reach school age, here are some questions you can use to make them think geometrically:

- How many sides does an octahedron have? By asking a child how many sides are on an octagon and how many legs an octopus has, they might discover that they can make a good guess. Show them the triangular openings as they appear on the balloon model. Does what you see match your guess?

- Even though the octahedron is covered in triangles, it has square cross sections. Show how the edges of the octahedron can be seen as three squares put together. If all the edges can be made by putting together three squares, how many edges does the octahedron have?

- If the octahedron is made from eight triangles with an edge on each side, and eight times three is 24, why does it have 12 edges, not 24?

- Can you use this same concept to correctly compute the number of vertices? Does this number match up with what you see on the model?

Don't forget to warn them to use their magic octahedron wisely, because knowledge is power!

Classroom Activities

Exploring geometry. Models such as the octahedron described in the first section can be used to explore geometry in a hands-on way. The Platonic solids are a good place to start. The simplest platonic solid, meaning the one with the least number of sides, is the tetrahedron. If you wanted to only traverse each edge once, it cannot be made out of only one balloon! Figure 3 shows how to construct it using two.

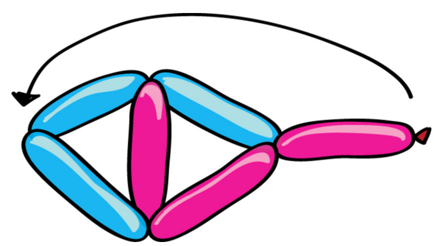

Figure 3: *How to make a balloon tetrahedron.*

Have students work out methods for building the other Platonic solids with as few balloons as possible. Using a solid model or picture as reference, can they figure out how many edges it has and how many balloons it would take? Into how many sections must they twist each balloon?

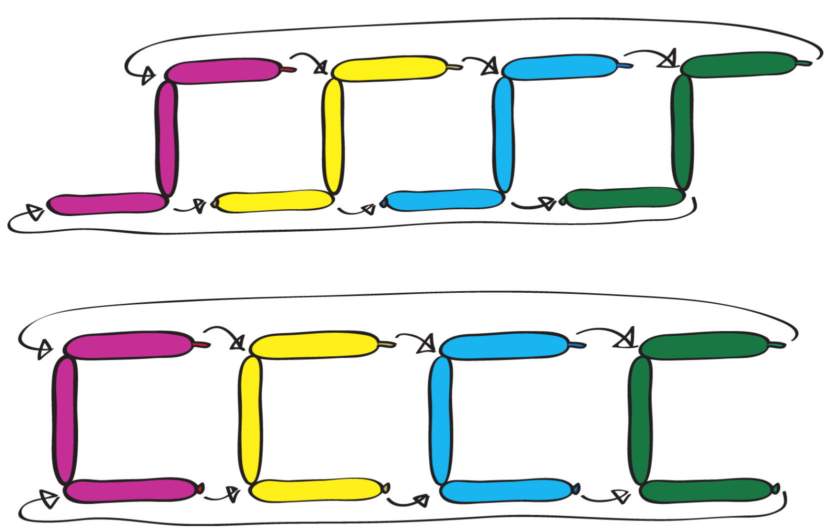

Figure 4: *Different methods for making a balloon cube.*

A fun puzzle is to design symmetric and aesthetically pleasing ways to link up the balloons. Challenge students to find different ways of putting together polyhedra. For example, the pieces of the cube can be linked up two distinct ways, as shown in figure 4, but one of those ways is chiral, so you can imagine putting it together so that either S or Z shapes are shown on the outside. The the two-balloon tetrahedron is also a chiral arrangement. Have students prove that there is no other way to make it. Figure 5 shows just one way of constructing the dodecahedron and icosahedron. Can students find other ways? Other symmetric ways?

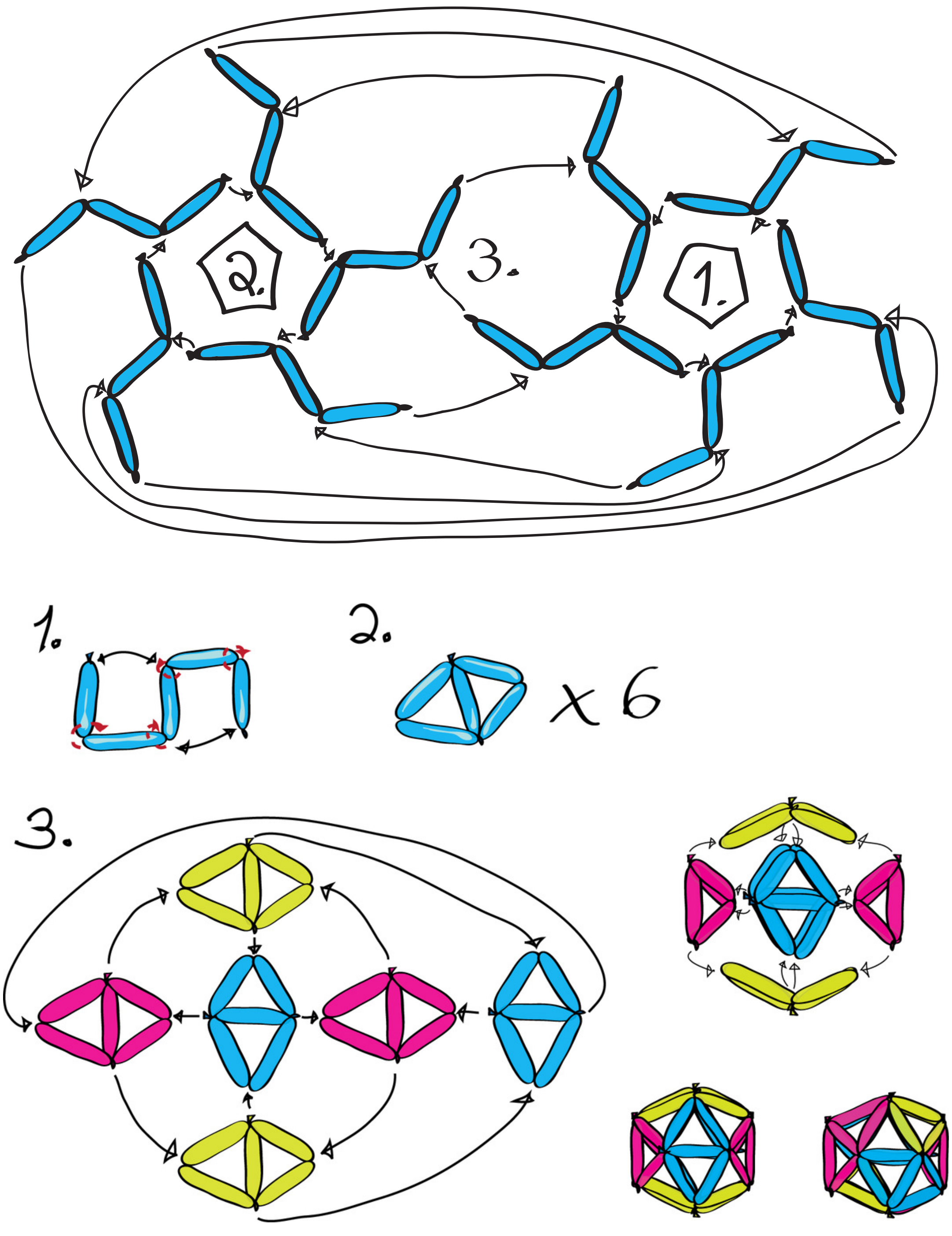

Figure 5: *One way to make a balloon dodecahedron and icosahedron.*

The same units that are used to make the icosahedron in Figure 5 can also be used to make any snub polyhedron. Each student can make one module, using an inflated balloon as a guide to make sure all students inflate their balloons to the same length. Then they can then work as a group to create larger polyhedra, such as the snub cube and snub dodecahedron in Figure 6.

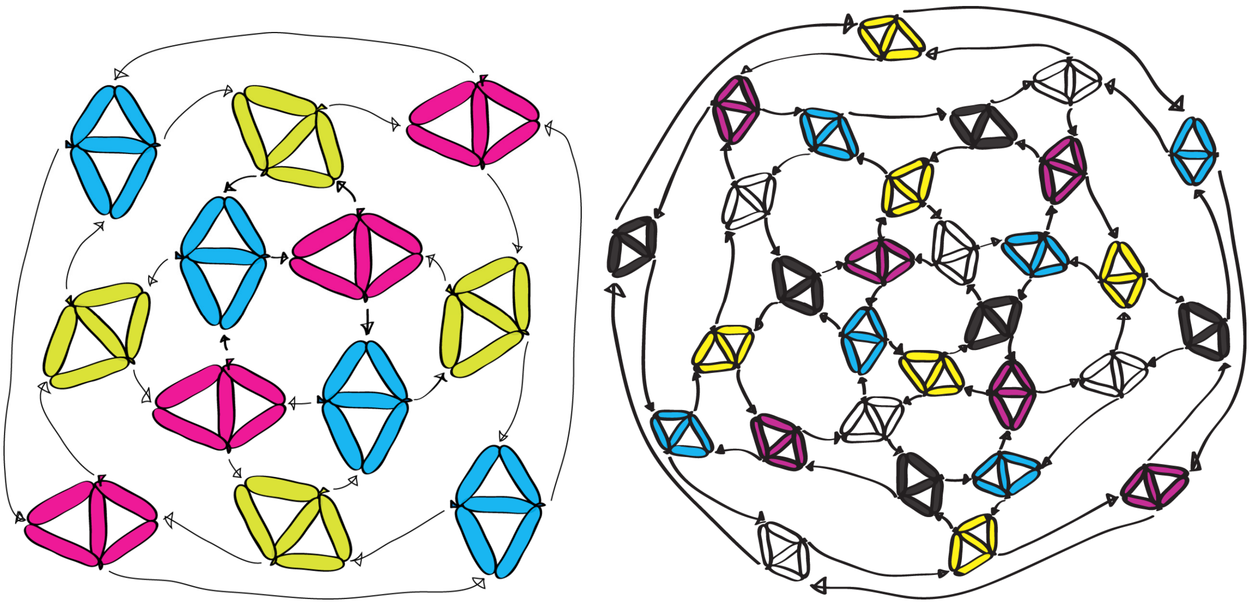

Figure 6: *How to combine 12 identical units into a snub cube, or 30 into a snub dodecahedron.*

There are an infinite number of polyhedra that are theoretically possible to make out of one balloon, including all dipyramids and antiprisms, though there are practical limitations. The octahedron is both a (square) dipyramid and a (triangular) antiprism. The triangular dipyramid is a bit simpler to make, as seen in Figure 7.

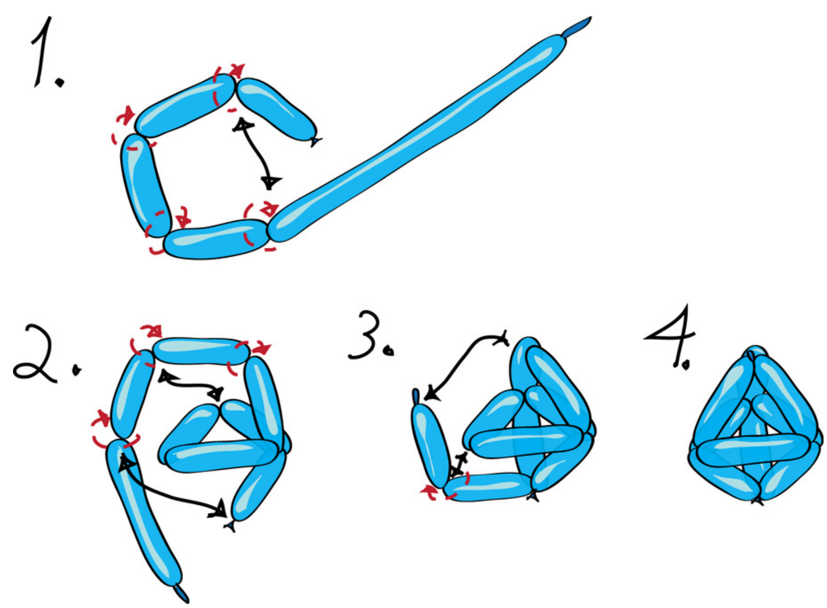

Figure 7: *Triangular dipyramid, out of one balloon.*

An introduction to graphs. A balloon sculpture can be thought of as a graph, as seen in Figure 8. Have students figure out how to draw the graph of a 3D model. In what different ways can you draw it?

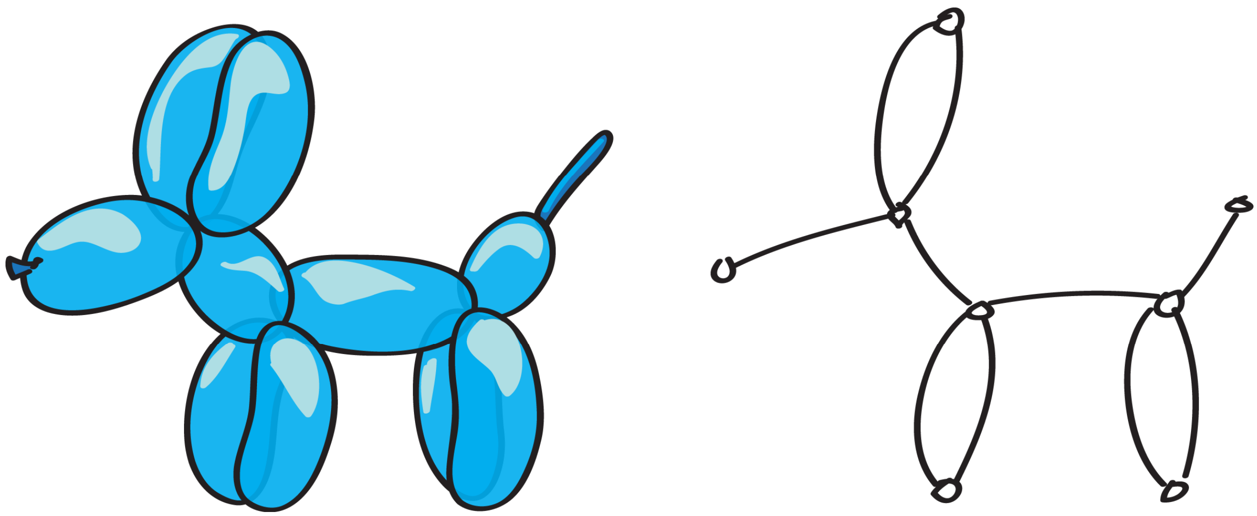

Figure 8: *A balloon dog and its graph.*

Instruct students to try making different balloon models from the same graph. Changing the lengths of edges and how the form is embedded in 3D can help students understand the essence of what a graph is, and how objects that look very different from one another can have the same structure. The balloon dog above would have the same graph if the body were long like a dachshund, or if its legs were different sizes, or sticking out opposite sides, or even if they were large loops and linked with each other.

Have students draw their own graph, and then figure out how to make it with balloons. How many balloons do they need? Can they figure out a rule that will let them know the minimum number of balloons required to make a given graph? Through experimentation, they may discover that they need to put the end of a balloon at any vertex where an odd number of edges meet. Because each balloon has exactly two ends, the minimum requirement for number of balloons equals the total number of vertices with odd degree, divided by two.

What kind of graphs can you make from just one balloon? This leads to the concept of Eulerian paths and cycles. This topic, as well as more difficult questions of balloon graph decomposition, are explored in [7].

Advanced Models

This section exists to give you a quick taste of what else is possible in the realm of mathematical balloon twisting. There are an infinite number of polyhedra, providing an infinite number of challenges, but beyond polyhedra there is a whole world of polytopes—the analogue of polyhedra but in different dimensions—and other mathematical figures are possible as well, such as crystal lattices, regular polylinks, and knots.

The 4-simplex is the four-dimensional analog of the tetrahedron, and is lovely when made out of a single balloon. This model is difficult because it uses two different edge lengths—six long edges make up the outer tetrahedron and four short edges meet at a central vertex.

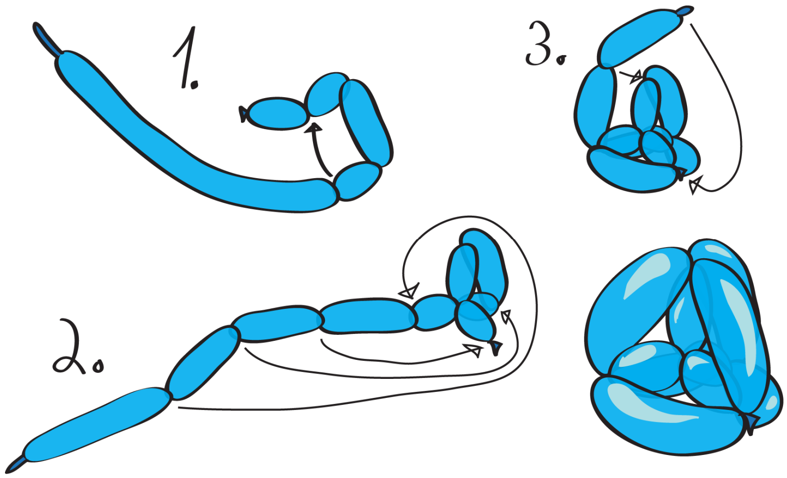

Figure 9: *4-simplex, out of one balloon.*

Regular polylinks, also known as “orderly tangles,” are a great challenge to weave together. Figure 10 shows how to weave together six squares into a tangle.

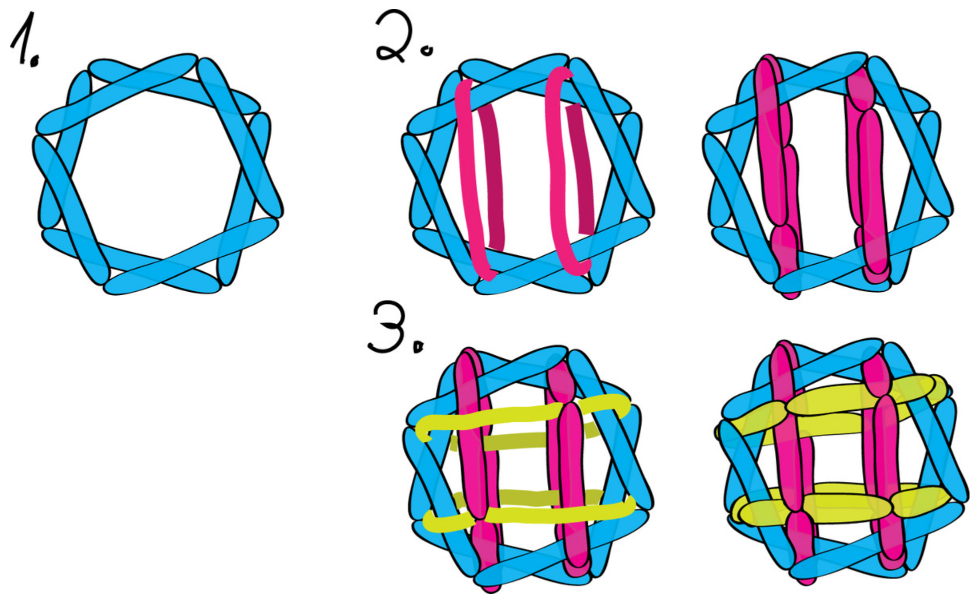

Figure 10: *How to weave the tangle of six squares.*

Figure 11 shows the finished tangle of six squares, simple Borromean rings made from three rectangles, a tangle of three triangles, and a tangle of five pentagons.

When making tangles, the ratio of length to width must be correct to get a snug fit. It is better to have balloons that are too long and thin to fit snugly, and then you can grab each corner and twist off a bit to create the right length edge, as seen in three of the models in Figure 11. Depending on the model, you may have to use 160s, such as with the tangle of five pentagons. 260s work well for the other three. Figuring out the exact length/width ratio for a perfect fit is another interesting question, explored in [8].

There are many more possible tangles! A good reference is [9]. More general material can be found on my mathematical balloon twisting webpage: http://vihart.com/balloons

Figure 11: *Tangles: three rectangles, six squares, four triangles, and five pentagons.*

References

[1] Erik D. Demaine, Martin L. Demaine, and Vi Hart, "Balloon Polyhedra", in *Shaping Space: A Polyhedral Approach*, edited by Marjorie Senechal and George Fleck, Second Edition, to appear.

[2] Jim Morey. YouTube: theAmatour's channel. http://www.youtube.com/user/theAmatour.

[3] Mishel Sabbah. Polyhedrons: Snub dodecahedron and snub truncated icosahedron. Balloon Animal Forum posting, March 7, 2008. http://www.balloon-animals.com/forum/index.php?topic=931.0. YouTube video: http://www.youtube.com/watch?v=Hf4tfoZC_L4.

[4] Jeremy Shafer. Icosahedron balloon ball. *Bay Area Rapid Folders Newsletter*, Summer/Fall 2007.

[5] Carl D. Worth. Balloon twisting. http://www.cworth.org/balloon twisting/, June 5, 2007.

[6] Rolf Eckhardt, Asif Karim, and Marcus Rehbein. Balloon molecules. http://www.balloonmolecules.com/

[7] Erik D. Demaine, Martin L. Demaine, and Vi Hart, "Computational Balloon Twisting: The Theory of Balloon Polyhedra", in *Proceedings of the 20th Canadian Conference on Computational Geometry (CCCG 2008)*, Montréal, Québec, Canada, August 2008.

[8] George W. Hart. Orderly tangles revisited. In *Mathematical Wizardry for a Gardner*, pages 187–210, 2009. http://www.georgehart.com/orderly-tangles-revisited/tangles.htm.

[9] Alan Holden, *Orderly Tangles: Cloverleafs, Gordian Knots, and Regular Polylinks*, Columbia U., 1983.