Tile Color Matching Using Simple Universal Cycles

Anna Virágvölgyi

Síp u. 6. Budapest H-1075, HUNGARY
E-mail: viragvolgyi.anna@gmail.com

Abstract

In a square tiling, one can mark squares using edge-colored matching rules. I describe a set of matching rules based on universal cycles. These arise when one studies arrangements of different letters from a small alphabet into a single sequence in which all possible permutation of a given length can be found. The results are interesting visually. They may have applications in creating parquet or other two dimensional tiling patterns.

Elements

Consider an alphabet of k letters, where k (k>2) is always odd. Using this alphabet, create a set of words where each word is of even length $2n$, but where no adjacent letters in the word are the same. For each n, if we ignore the direction of reading, the number of possible words is $S_n=k(k-1)^{2(n-1)}$.

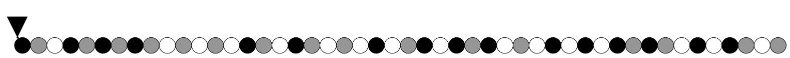
A universal cycle is a compact listing of a class of combinatorial objects [1]. One can prove that for the above sets of words exist universal cycles. An unwrapped universal cycle for n=3, $S_3=3\times2^4=48$ with alphabet {**a**, **b**, **c**}:

a b c a b a b a b c b c b c a b c a b c b c a c b a c a b a c b c a c a c a b a b c a c a b c b

This cyclic string contains **a**, **b** and **c** equally 16 times. Each above defined word of length 6 occurs exactly once on this cycle:

a b a b a c
b a b a c a
a b a c a c ... etc.

Each letter (**a**, **b**, **c**) occurs in the entire set of words the same number of times – $2n(k-1)^{2n-2}$ = 2x3x2^4 = 96. With other symbols (**a** = ●, **b** = ●, **c** = ○) the above chain is:



By substituting stripes for beads due to of the nature of universal cycles each elements of S_3 one can get as diagonal striped square tiles.

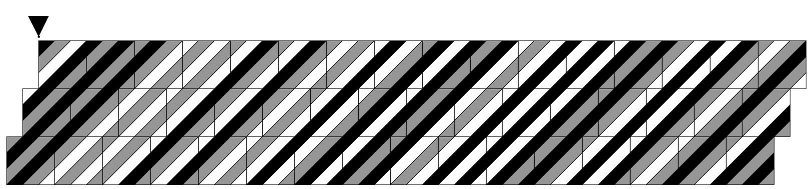


Figure 1: *Unwrapped universal cycle with the S_3 =48 elements.*

Areas of the different colors in the **Figure 1** are equal to each other. The picture shows how a great number of possible interconnection are between this tiles. This feature enable the tiles to be matched in many ways. The rotated tiles can be matched as well.

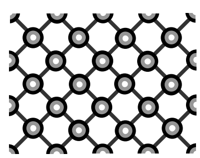
Patterns

a b c

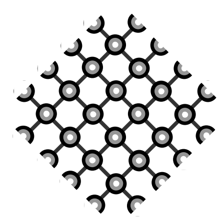
Figure 2

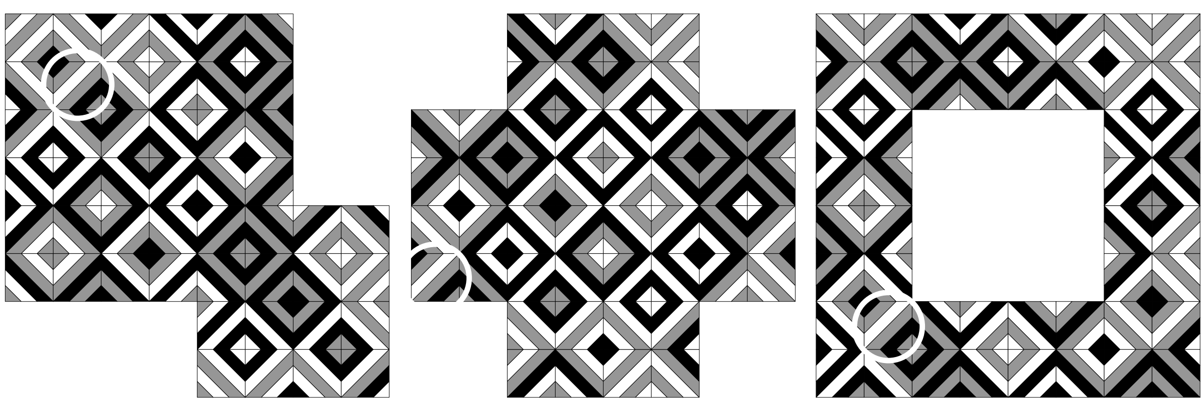
Each half-tile can fit seven ($2^3 - 1$) others. Tiles in an arrangement are different and none of them earn distinction. Nevertheless the form of arrangements contiguous to the position of the tile.
The white circles show localization of a certain tile in different arrangements. A tile's rotation and the combination of the eight fitted tiles (framed with dark line on **c** picture of **Figure 2**) determine the form of the pattern.

The number of the edges of tiles on the circumferences of the pictures are different. **Figure 2**'s arrangements have less exterior and more interior tiles than **Figure 3**'s ones. The structures of the **b** pictures in **Figure 2** and **3** are on the right .



On the left the rectangular arrangement has 17 "eyes" (= ◉), while in the cross (structure on the right) are only 16 ones.





a b c

Figure 3

In spite of no square is identical to any others and there are no special symmetries or groups in the pattern of pictures the form of pictures are symmetrical.

Reference

[1] G. Brockman, B. Kay, E. E. Snively, On Universal Cycles of Labeled Graphs, *The Electronic Journal of Combinatorics* 17 (1), r4. <http://www.combinatorics.org/Volume_17/PDF/v17i1r4.pdf>