Bridges 2010: Mathematics, Music, Art, Architecture, Culture

Constructing Sierpinski Triangle with Rings

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Abstract

This paper is going to present an artistic arrangement of circular rings that form the Sierpinski Triangle in a different way. In this simple way, it does not matter which initial form or initiator we start to work with. What is so important in construction of an accurate Sierpinski Triangle is the arrangement of the shapes on the plane. Thus, it is our generator which always must be deterministic in different scales, in this case.

Introduction

Most of the prevalent ways to reach Sierpinski pattern, with different amounts of deviation or approximation, start with adding or discarding triangles as different processes of iteration. One of the very different ones, more of interest to me, is Sierpinski Arrowhead [1]. However, it only presents the boundary. This interesting curve starts with Koch curve and the more we repeat the iteration, the clearer does the revealed Sierpinski pattern get. Such mixed patterns are more interesting because nature behaves mystifyingly and complicatedly. Nature does not purely choose the patterns of the fractals it is going to launch in different situations. It behaves chaotic and uses any pattern and material available to present its intention, somehow different from pure mathematics.

Here, I want to start with circular rings and at the end provide a Sierpinski triangle, not a curve. The most important thing is that in this process we do not cut or discard anything and only add rings according to our main and deterministic iteration pattern.

Our construction starts with three completely circular rings crossing the central points of each other. This way, there will be provided a triangular common area at the centre (Figure 1). In the second iteration we provide 3 similar sets of rings with circles that have diameters half of those of the basic circles of the first iteration. We put these sets on the plane so that the central points of their common triangular areas coincide with the central points of the circles belonging to previous iteration (Figure 2). This process continues similarly in next levels. During the first six iterations (Figures 1-6), it is not clear to which final pattern we are attaining. From the seventh (Figure 7), where the boundaries of the circles start to cover some areas, it gets clearer that a Sierpinski triangle is appearing and at the tenth iteration (Figure 9) it gets completely evident. The interesting part is that although we only have rings, we attain a Sierpinski triangle constructed of numerous triangles. The most important phenomenon is how the boundaries of these circles cover some triangles just like those of original pattern of Sierpinski triangle.

On the other hand, if you watch the outer borders of covered triangles and inside the holes, you will face a new different clustering fractal pattern. In figure 10 where the thickness of the rings is decreased and the number of iterations is increased to fifteen, the resulted Sierpinski triangle looks more adequate. Fundamentally, the less the thickness of the rings and the more the number of iterations gets, the more adequately the final image will come out. The number of the rings in each iteration can be calculated according to the formula N(number of rings) = $(3^{n+1}-3)/2$ in which n is the respective number of each certain iteration.

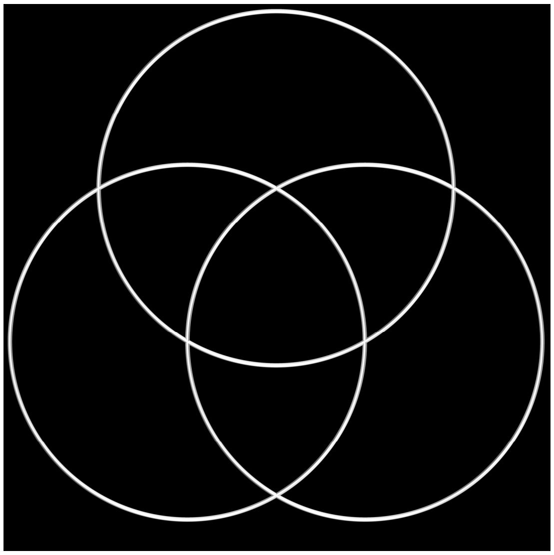

Figure 1: *The first step is to draw three equal circles crossing the central points of each other. At the center of the plane there will be constructed an equilaterally triangular shape made by three arcs. (Iteration 1 with 3 rings)*

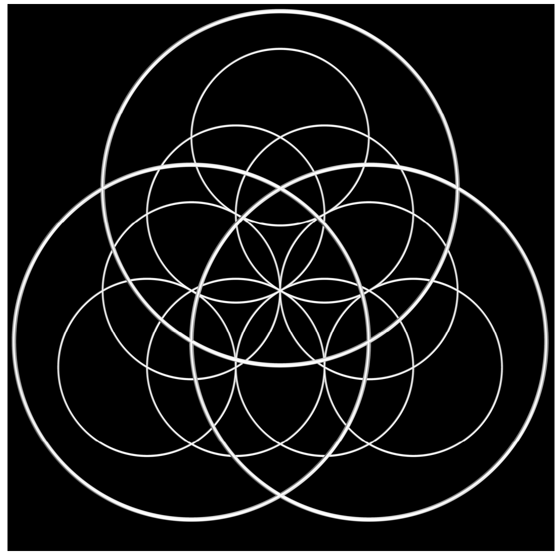

Figure 2: *We provide three other distinct sets of what we had in previous process and put them on the plane so that the central points of their central triangular shapes and three vertices of the previous iteration's triangular center (The central points of the previous iteration's circles) coincide. (Iteration 2 with 12 rings)*

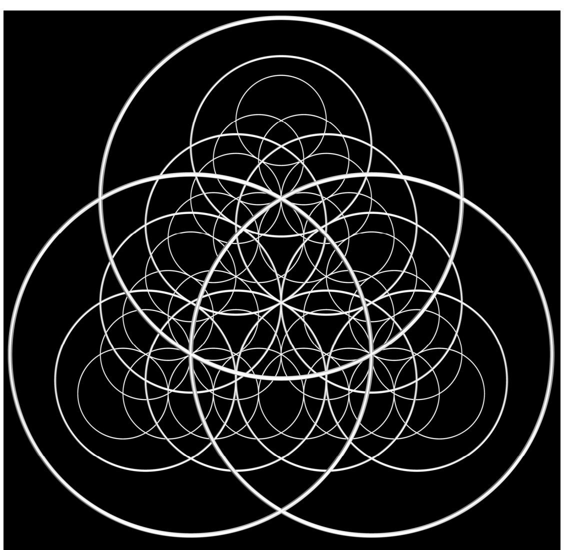

Figure 3: *Now, that our initiator and generator have got clearly disclosed, we act completely repetitive to reach higher iterations. (Iteration 3 with 39 rings)*

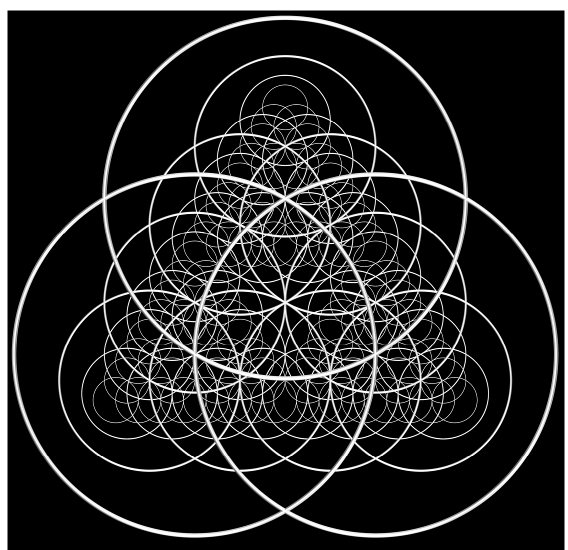

Figure 4: *Iteration 4 with 120 rings*

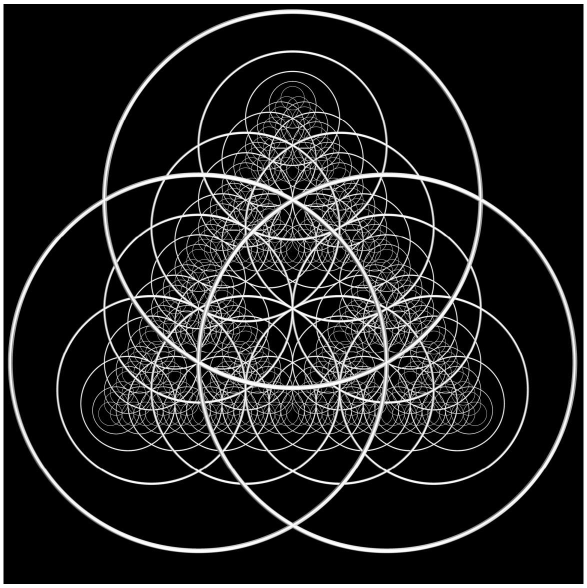

Figure 5: *Iteration 5 with 363 rings*

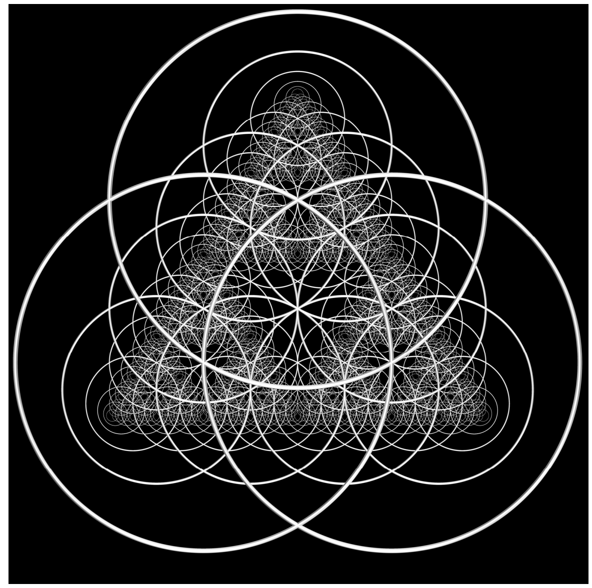

Figure 6: *Iteration 6 with 1092 rings*

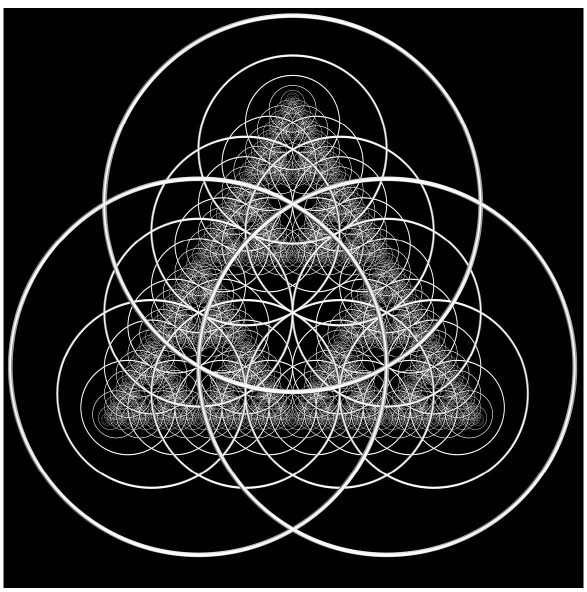

Figure 7: *Henceforth, it gets clear which final fractal pattern we are attaining. (Iteration 7 with 3279 rings)*

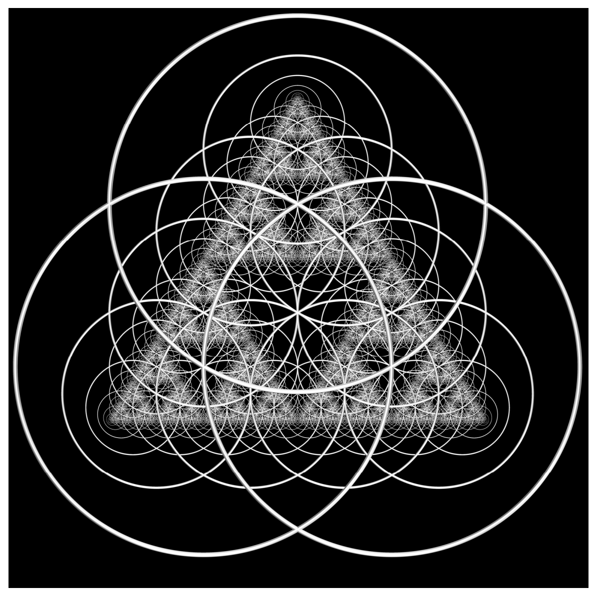

Figure 8: *The more we go ahead, the more the concentration of the rings, on the areas which should be covered in original Sierpinski triangle, gets. (Iteration 8 with 9840 rings)*

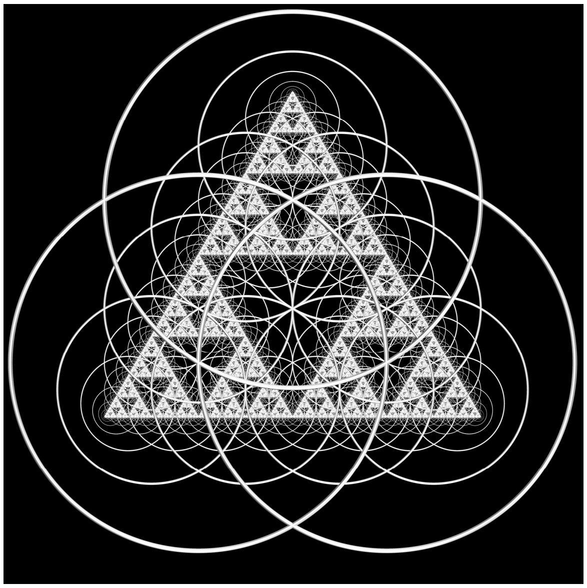

Figure 9: *Now, after ten iterations the Sierpinski pattern has got revealed completely. (Iteration 10 with 88572 rings)*

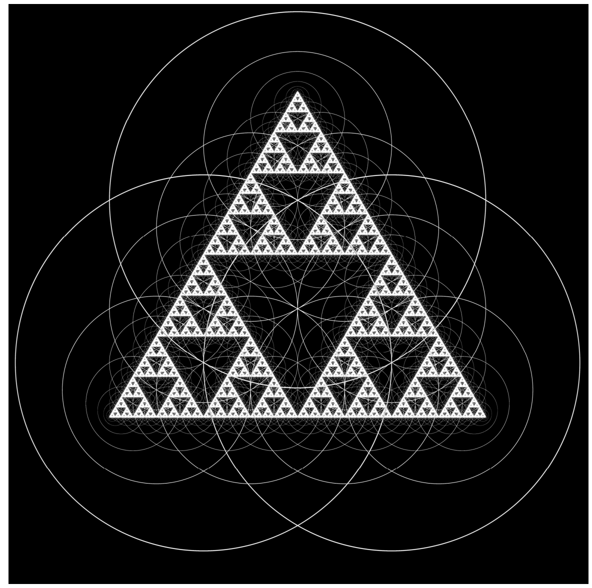

Figure 10: *In this image, I have decreased the thickness of the rings to one fifth and have increased the iteration number to fifteen, in order to have a clearer result. (Iteration 15 with 21523359 rings)*

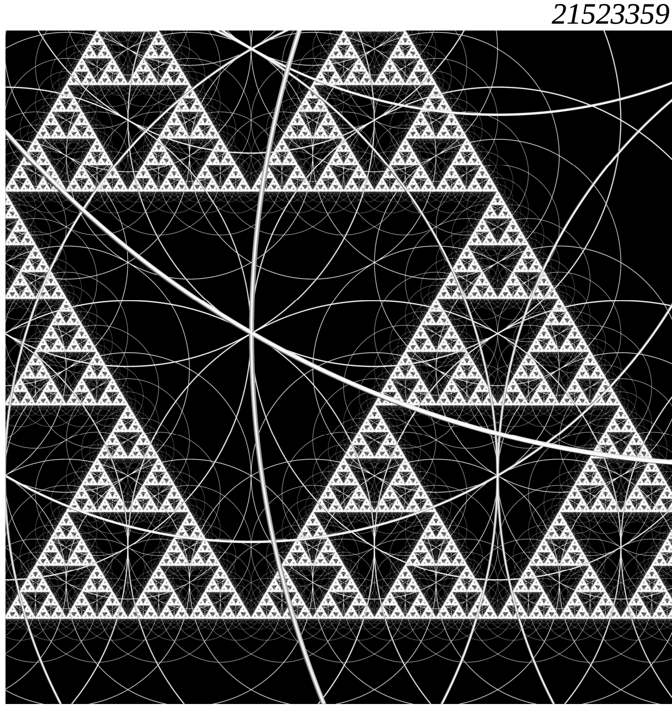

Figure 11: *A closer look at Figure 10 with a magnification of 4X .*

Reference

[1] Mandelbrot, B.B., 1983, *The Fractal Geometry of Nature*, p.142. W. H. Freeman, NY.