From the Angle of Quasicrystals

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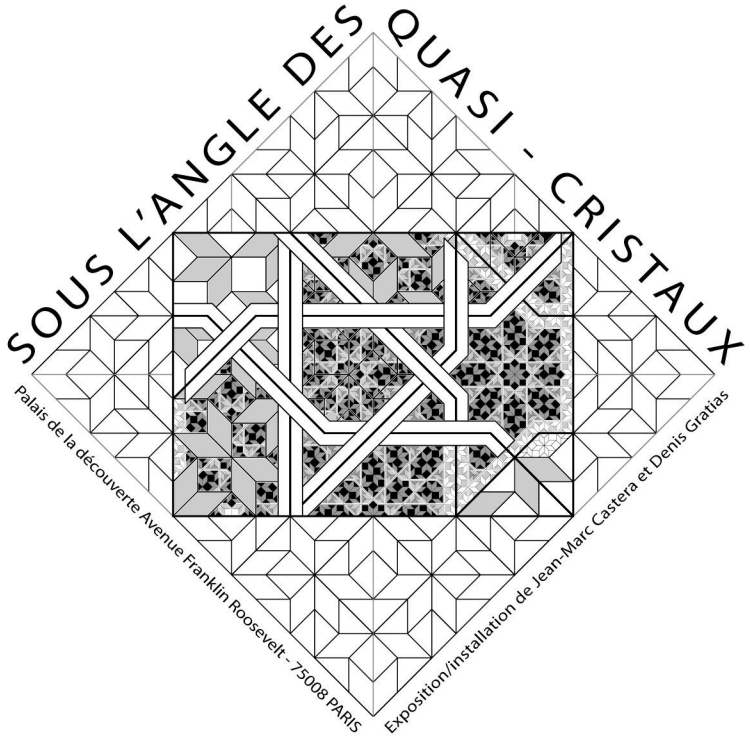

Figure 1: *Poster of the exhibition.*

Abstract

Opened in December 2009 and dedicated to quasicrystals and their application in art and architecture, "From the Angle of Quasicrystals" is a permanent exhibition/installation at the "Palais de la découverte" museum in Paris. The physicist Denis Gratias, one of the most prominent theoreticians in quasicrystals, was in charge of the "pure scientific" part, while I was to be the artist of the band.
A spot located between the planetarium and the Pi-room was put at our disposal. We wanted this space to be entirely structured by a unique quasicrystalline pattern, visible at different scales.
This pattern covers continuously two adjacent walls and the floor. A display cabinet showing real quasicrystals is inserted at an angle in accordance with the pattern, and two custom pieces of furniture are designed and placed, also matching the structure. One piece of furniture is used as table for a kind of puzzle especially made for this installation, intended for visitors to play with, and the other includes a display screen for an interactive software program and slide show with my applications of quasicrystals to architecture.

1. Starting Point

The “Palais de la découverte” (www.palais-decouverte.fr) is the oldest science museum in Paris. Aside from providing permanent installations and temporary exhibitions, its mission is to offer daily talks, demonstrations and interactive exhibits. Our installation is located in the math space, between the planetarium and the famous round-shaped room dedicated to the number Pi.
We wanted the space to be attractive, and to enhance the concept of relationship, therefore all elements are connected to a unique quasicrystalline structure.
In order to keep the objective simple, we started with an octagonal (2D) quasicrystal, made from the inflation process of an Ammann-Beenker aperiodic tiling. This is the place for $\sqrt{2}$.

2. The Main Pattern

2.1. Walls and floor. We have been quite lucky throughout the whole process, as every parameter fit very well with the scheme design. The pattern is made up of squares and diamonds, and all angles are multiples of 45°. The floor reflects the same pattern in continuity with the walls (fig. 6).The use of 6 levels of inflation (fig.2) competes the graphic palette, and each vertex is accentuated by a stud.

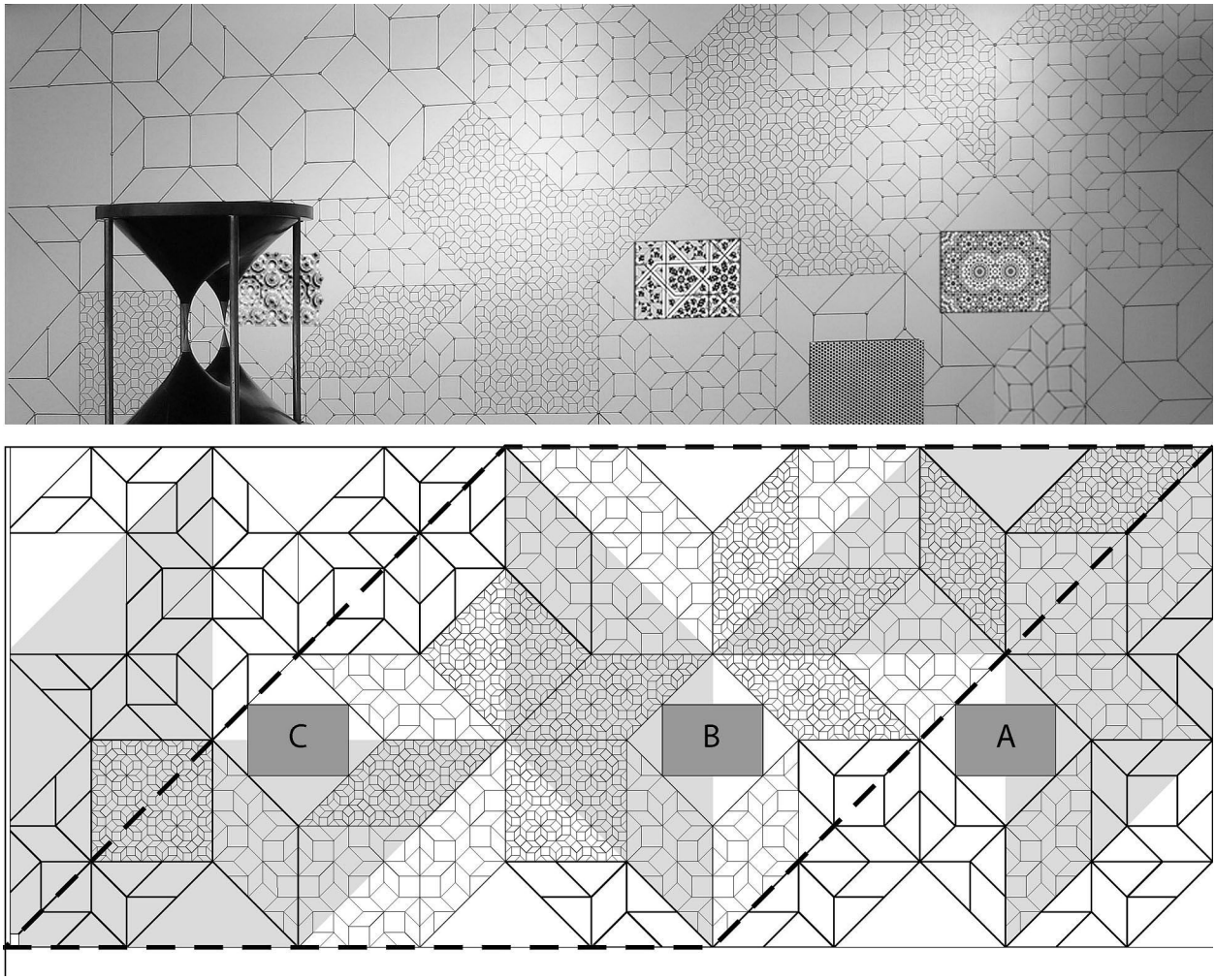

Figure 2. ***Top*** *: part of the finished wall.* ***Bottom*** *: scheme design, with 6 inflation levels. The dotted lines denote the first level. Smaller lines are level six. A,B, C show the panel's place.*

3. The Panels

Three panels show three links between quasicrystals and traditional Arabic geometric art : through the overall structure of Zellij (ceramic mosaic, [5]), the individual Zellij tiles, and the muqarnas structures.

3.1 Panel A : skeleton level (fig. 3). This is the first case of a connection between the geometric arabesque (octagonal family) and quasicrystals that arose in our work [4]. Generally at the beginning of a Zellij design there is an underlying structure we call the skeleton, made up of the alternation of two tiles : the octagonal star and an irregular hexagon called "Saft". The point is that the skeleton follows the outline of an isometric network similar to an octagonal quasicrystal. If it doesn't match perfectly, we definitely can still draw a Zellij skeleton from such a quasicrystal. The same remark works for a map of muqarnas.
Since the entire skeleton of our panel had to match the pattern of the wall (fig. 3, bottom), it became impossible to get a traditional Zellij motif, with a central rosette. Then comes a motif with two overlapping rosettes, which can be called a double 16 folded star. Conceived as a double star (behind this wall is the planetarium !), we can see the trajectory of a planet running around one star then around the other, drawing the sign of the infinite and/or the number 8, both main characters of the installation (fig.3, bottom).
Excepting the "owl's eyes", centers of the 16-folded rosettes, every single tile belongs to the Zellij basic repertory and each angle is a multiple of 45°.

3.2 Panel B : Tiles, interlaces, and sub-tiles level (fig. 4). This panel, drawn according to the overall structure (see fig.1) express two things, which are linked.
First, a self-similarity potential : the large Zellij tiles, and the interlaces as well, can be broken down into similar Zellij tiles on a smaller scale [3].
Then, we have a new link with octagonal quasicrystals : each large Zellij tile is made up from squares (or half squares) and diamonds. The same property may be seen on a smaller scale, with the interlaces and second level of Zellij tiles [2]. See also part 6, fig.7 and fig.8 left.
The "Abyme puzzle", which the public is welcome to manipulate, is an application of these properties.

3.3 Panel C : Connection with muqarnas structures (fig. 5). Muqarnas, or stalactites, are complex-looking 3D structures used in architecture as friezes, arcs, corbels, the most amazing applications being to domes ([3],[5]). They are mostly made up of modules in wood or plaster. Our drawing uses only the four main distinct modules. It is drawn from the same quasicrystalline pattern as are the other two panels, and is repeated in order to provide a stereoscopic point of view (people are encouraged to look at the drawing with crossed eyes until the two centers overlap and the 3D effect emerges).

4. The Display Cabinet

The cabinet shape is the quarter part of an octagon, again fitting with the overall pattern. Inside is a rare specimen of a real quasicrystal, together with a text written by Denis Gratias, talking about the amazing adventure of this discovery, with some historical photographs showing the pentagonal local symmetry. This is the place for Number five and the golden section.

5. The Screen Display

The slide show plays some of my animations and design applications of quasicrystals in architecture ([2]), and an interactive program written by Patrick Rietmacker.

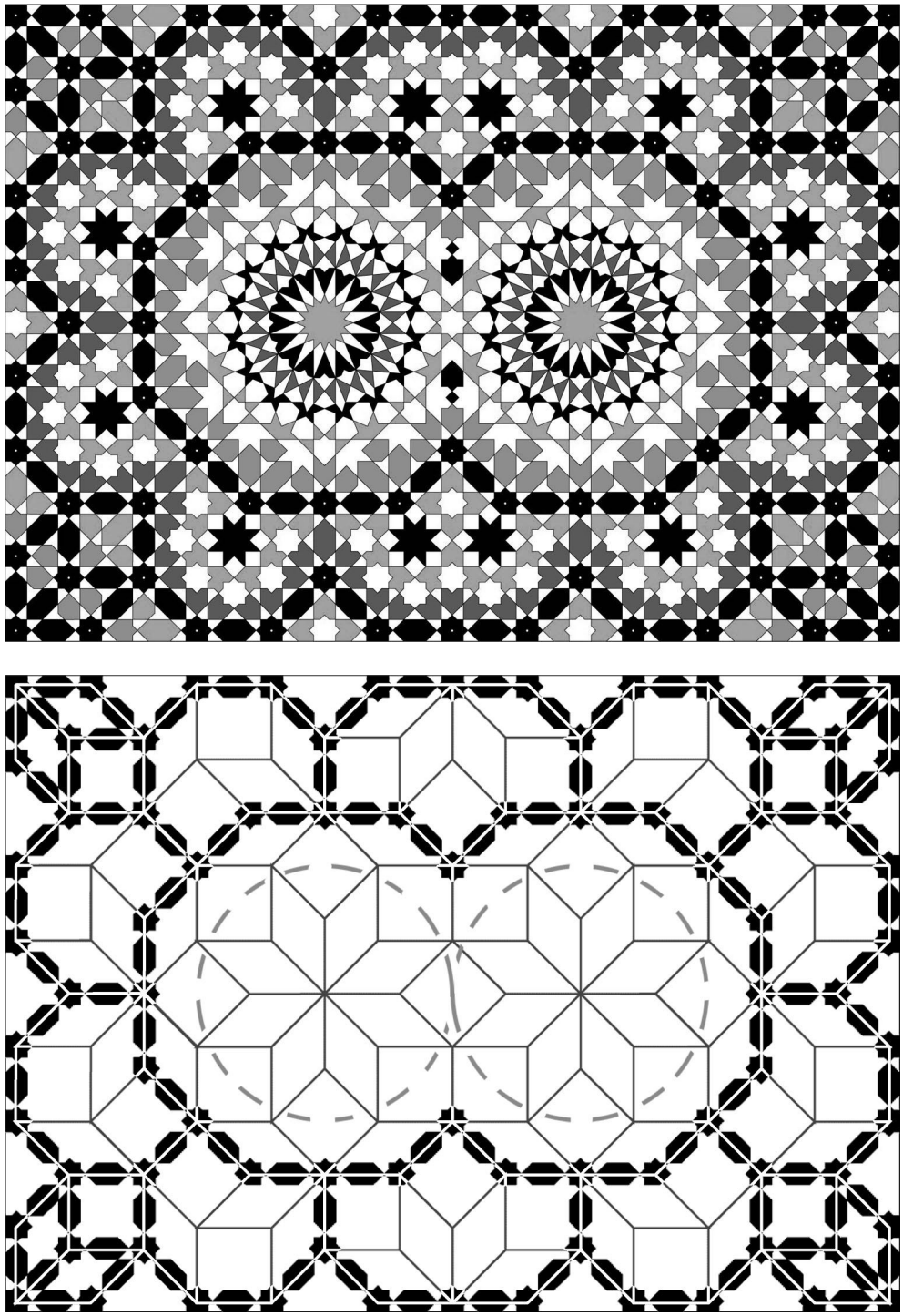

Figure 3: ***Top****, a Zellij style motif.*
Bottom*, the skeleton, drawn from the smallest quasicrystalline pattern of the wall.*
Different Zellij motifs can be draw from the same skeleton.
More explanations in [4] and [5].

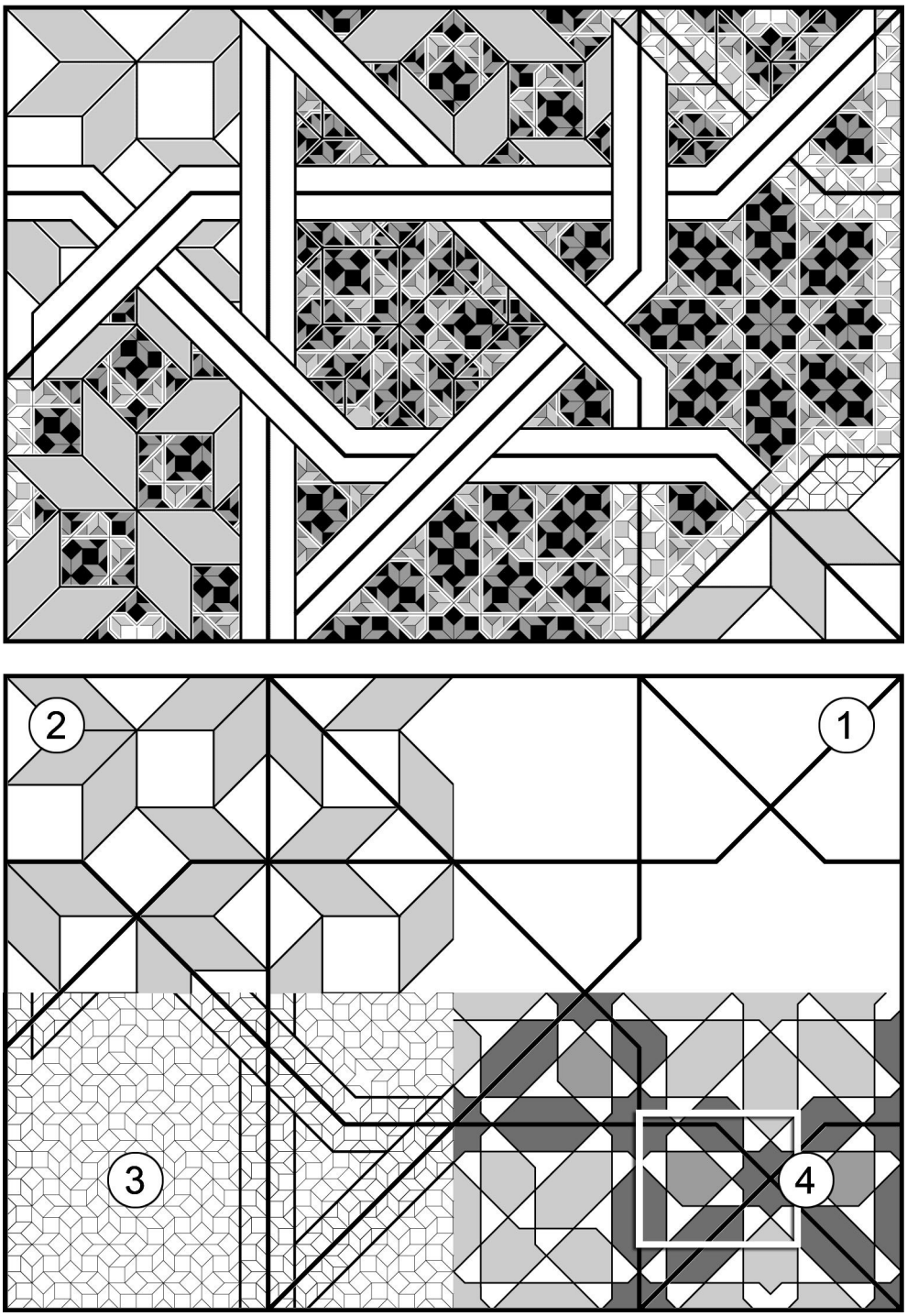

Figure 4: ***On top,*** *the panel drawn to match the pattern of the wall (see fig.1) .*

On bottom (1): *The high level Zellij pattern*. (2): *Subdivision of the tiles. They fit together as a diamond/squares pattern, potentially an octagonal quasicrystal. (3): Pattern with interlaces are broken down into small diamonds and squares, according to the second level of Zellij tiles* (4)*, where we can see a reduced copy of the initial Zellij drawing. See also fig.7 and fig.8, left.*
The process can go on indefinitely.

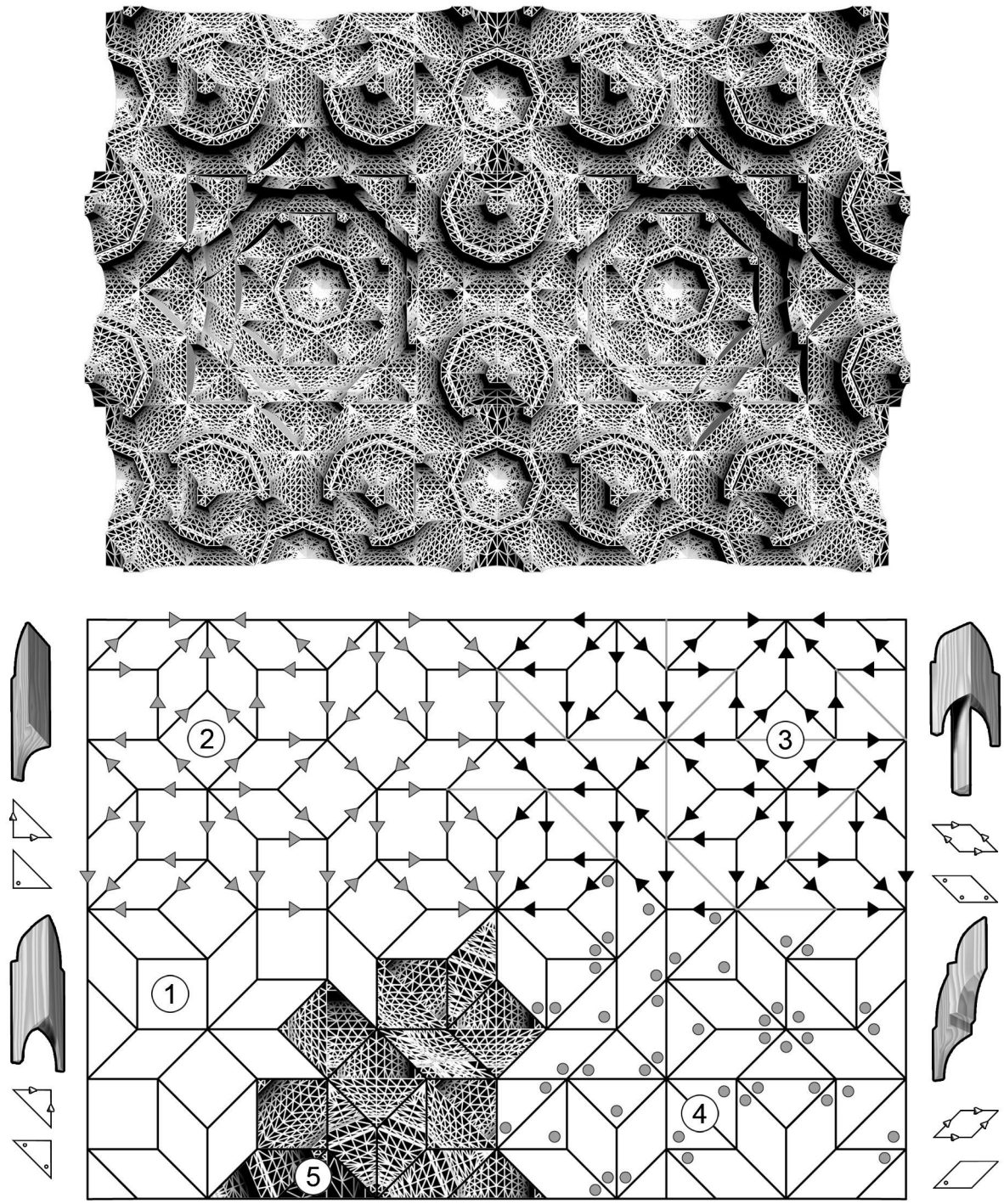

Figure 5: ***On top****, the muqarnas structure, drawn according to the quasicrystalline pattern of the wall. This drawing can be considered as a stereoscopic pair, and viewed in 3D.*

Bottom*, scheme design.* (1): *Original square-diamond structure, part of the octagonal quasicrystal.* (2): *Orientation of the edges.* (3): *Division of the squares into halves*. (4): S*tandard coding for the muqarnas units.* (5): *CAD drawing, in a wire frame rendering.*
Depending on the orientations of the edges, different muqarnas structures can be draw from the same starting point. More information in [3] and [5].

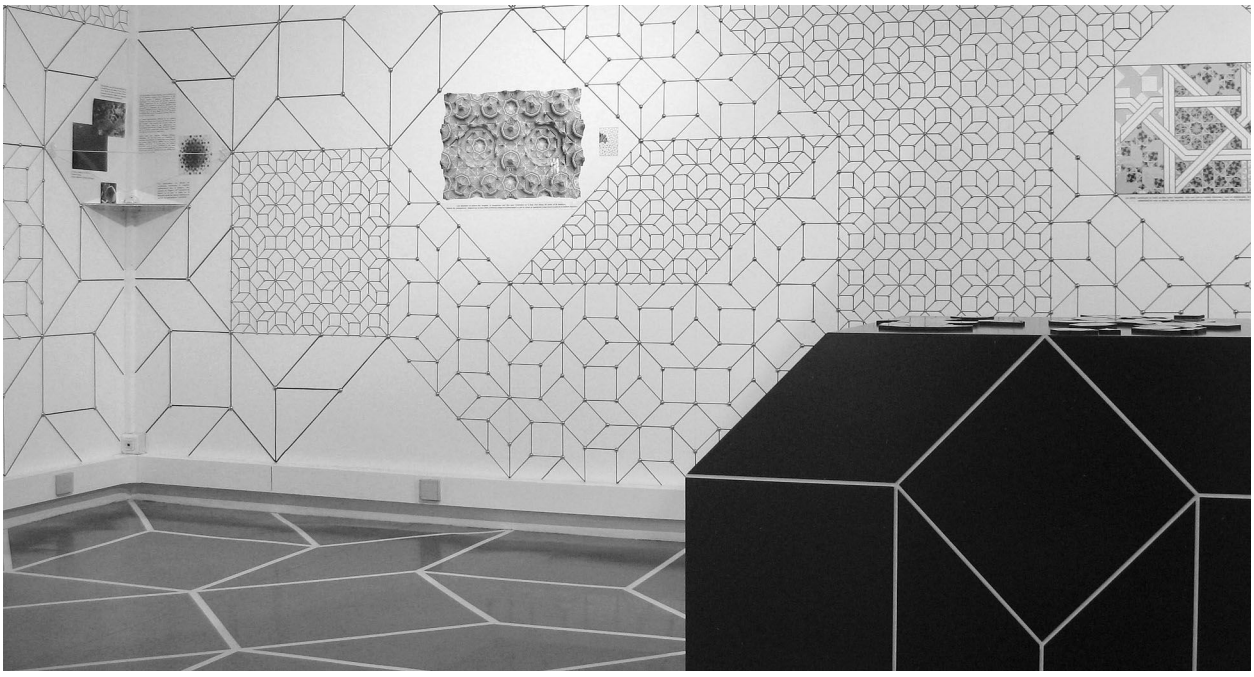

Figure 6: *In the angle, the display cabinet inserted in the wall pattern. Right, the black furniture for the puzzle, set up to match the floor pattern.*

6. The "Abyme"Puzzle

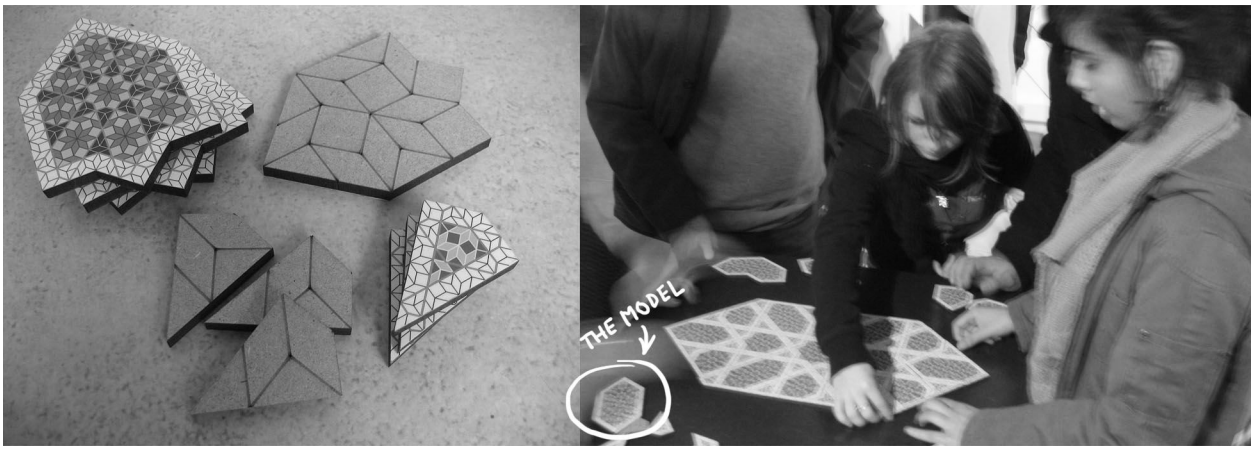

Figure 7: ***Left*** *: some tiles, recto and verso.* ***Right*** *: kids in action.*

This game uses a set of laser-cut tiles whose shapes belong to the basic repertory of Zellij. On the verso is laser-engraved a subdivision into diamonds, squares and half squares. When the tiles are put together following the rules of Zellij, the half squares turns into squares and we can get a quasicrystalline pattern.
According to the self-similarity property, the recto is decorated with smaller Zellij tiles, themselves decorated with squares and diamonds, on a smaller scale than the verso (fig. 4 and fig.7, left).
We can play the game the same way we play with standard (undecorated) Zellij tiles. But, if we say this is a puzzle, where is the picture we have to reconstitute ? Well, you hold it in your hands : any tile can be the model you have to build up on a larger scale (fig. 7, right).

7. Conclusion

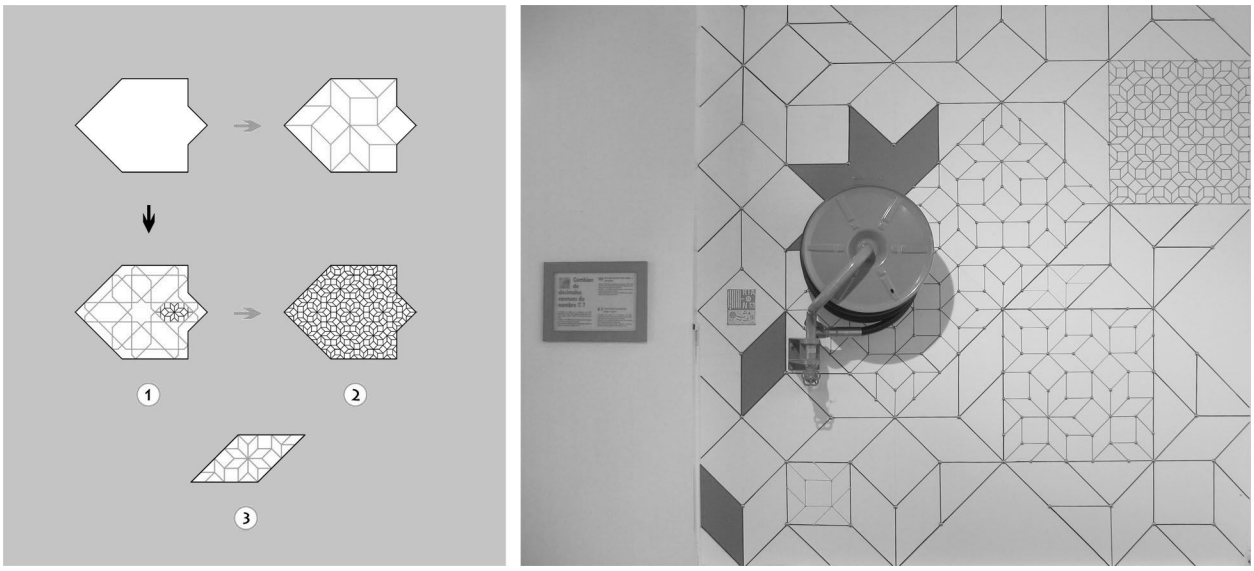

Figure 8: ***Left.*** *(1): Two levels of Zellij tiles; (2): Two levels of square/diamond , and (3): Their scale relationship. The scale factor is 1/2(√2 – 1).* ***Righ.*** *The fire hose is incorporated into the pattern.*

This installation reflects the idea of a world entirely ordered by a unique structure, with its specific rules. As we know that the world is always richer and more complex than anything we can imagine, this idea sounds a bit reductive. Fortunately, our installation avoid this issue: At the first visit to the site people said, talking about the red fire hose : "it's a pity, we cannot get rid of that ! ". But, finally, it turned out that the hose kindly accepted to be incorporated into the structure, even enriching it by its own characteristics (fig. 8, right): The color of the panels and of the puzzle tiles come from something that was first considered just as an undesirable element.
These last considerations belong to the artistic side of the experience.

References

[1] : Jay Bonner, 2003, *Three traditions of self-similarity in fourteenth and fifteenth century Islamic geometric ornament*, Meeting Alhambra, Proc. ISAMA-Bridges 2003, p. 1--12.
[2] : J.M. Castera, *Arabesques for Abu Dhabi*, Symmetry : Culture and Science Vol. 19, 2008.
[3] : J.M. Castera, *Play with infinity*, Meeting Alhambra, Proc. ISAMA-Bridges 2003, pp. 189-196, 2003.
[4] : J.M. Castera, H.Jolis, *Géométrie douce*, éditions $6^{1/2}$, Paris, 1992.
[5] : J.M. Castera, , *Arabesques*, ACR éditions, Paris, 1996 (French), 1999 (English).
[6] : D. Gratias, A. Katz, *Tiling and Quasicrystals, in Lectures on Quasicrystals,* Les éditions de physique, Les Ulis, 1994.
[7] : E. Makovicky, F. Rull Pérez and P. Fenoll Hach-Alí, *Mirador de Lindaraja: Islamic ornamental patterns based on octagonal lattice in Alhambra, Granada, and Alcazar, Sevilla, Spain*, Boletín Sociedad Espaňola Mineralogía 19 (1986), 1-26, 1992.
[8] : M. Senechal, *The Mysterious Mr. Ammann*, The Mathematical Intelligencer, 26:4, 2004.
[9] : D. Shechtman, I. Blech, D. Gratias et J. W. Cahn, *Metallic Phase with Long-Range Orientational Order and No Translational Symmetry*, Phys. Rev. Lett. 53, 1951-1953 (1984).