

Types of Repetition and Multistable Perception in the Frieze Patterns on the Marble Pavement of the Cathedral of Siena

Yang Liu
Independent Artist and Author
6 Pine Street
Belmont, MA
U.S.A. 02478
yangliu1971@gmail.com

Godfried Toussaint
Radcliffe Institute for Advanced Study
Harvard University
Byerly Hall, 10 Garden Street
Cambridge, MA, U.S.A. 02138
godfried@cs.mcgill.ca

Abstract

The marble pavement of the Cathedral in the Tuscan city of Siena in Italy has been described as one of the marvels of the world. Over the centuries much has been written about its biblical and political characters, the stories depicted in its figurative mosaics, the artists responsible for creating the mosaics, the types of marble used, and the history of their construction. The many frieze patterns framing the figurative mosaics are marvels of *geometric* design, and yet, they have been conspicuously overlooked in the vast literature concerning this pavement. Here these geometric frieze patterns are analyzed in terms of a typology of patterns of repetition, as well as from the point of view of the optical effects, such as multistable perception, that they engender in the viewer.

Introduction

The Cathedral of Siena in the Tuscan region of Italy is a wonder to behold from both the outside and inside. Built over a time-span of over six centuries, it is rich with history, and dear to the hearts of the Sienese people. The marble pavement inside the cathedral, over which people shuffle their feet is as stunning as the architecture of the building itself. When the Bishop of Perugia visited the cathedral in 1575, he remarked that “*the pavement is truly beyond belief*” [2]. Indeed, the Italian painter, architect, and art-historian Giorgio Vasari has described it as “*the most beautiful ... great and magnificent pavement ever made*” [1]. Over the centuries much has been written about its biblical and political characters, the stories depicted in its figurative mosaics, the types of marble used, the artists responsible for creating the mosaics, and the complex history of their design and construction [1]-[4]. The frieze patterns framing the figurative mosaics on the pavement are marvels of geometric design, and yet, in contrast to the figures they highlight, these geometric designs have been conspicuously overlooked in the literature dealing with this pavement. Books and articles describing, analyzing, and illustrating the mosaics routinely cut off these frieze patterns as if they are not important. A six hundred-page Ph.D. thesis on the pavement mentions the friezes but only in the context of the commissions, dates, and costs of the different color marbles requisitioned for their construction [2]. Bruno Santi [1], devotes only one sentence in his book to these marvelous geometric frieze patterns on page 37, where he states concerning the Presbytery and Choir section of the pavement: “*The portion of the panels between the portraits contains geometrical motifs which create exceptional optical effects.*” Here we fill these gaping holes present in the literature on these geometric frieze patterns: we analyze them in terms of a typology of patterns of repetition, and from the point of view of the *exceptional optical effects*, most notably multistable perception, that they produce in the viewer. Although the Siena Cathedral frieze patterns belong to the more general medieval Cosmati tradition of marble working [26]-[27], we restrict our analysis to the Cathedral of Siena pavement.

Repetition is a ubiquitous property of nature, science, visual arts, music, and life in general. The hallmark of the scientific method is repeatability of experiments [5]. A musical rhythm without repetition is not considered danceable. Practice makes perfect is a well-known idiom. Yet one often hears the word used in a pejorative manner: this music is too repetitive, my job is too tedious, I am tired of this routine, and so on. Analysis of the term however, rather quickly reveals that repetition plays a crucial role in our daily lives. It is after all a question of balance. In the words of the Danish philosopher Kierkegaard [12], “*Repetition is the reality and significance of life.*” Another philosopher, Heinemann [12] expresses the same thesis in different words: “*Origin and repetition are pervasive and correlative characteristics of the world as we know it. The two are interdependent and cannot be understood, except in their correlation.*”

The study of the nature and role of repetition in the visual arts is scant. Most papers that deal with repetition are in fact found in the music literature [9]-[11]. One exception is the paper by Toussaint and Liu [6] that surveys the different types of repetition studied in music, transfers them to the domain of the visual arts, and presents some new categories of repetition. Calvin S. Brown, Jr., proposed four types of repetition in music that were readily transferred to the analysis of Greek and Roman meander patterns in [6]. Brown’s four main categories are: *formal*, *intellectual*, *literary*, and *triumphant*. Brown’s definition of *formal repetition* implies the *exact* repetition of portions of the pattern. His definition of *intellectual repetition*, implies “sameness with difference,” approximation of repetition, similarity, or repetition with variation. *Literary repetition* refers to the repetition of a symbolic element, such as the *leitmotifs* in Wagner’s operas. Triumphant repetition is a very different category of repetition in that, unlike the others, it does not reflect any structure, geometric or otherwise. Rather it is a term Brown reserves for the type of repetition that elicits a tremendous emotional response, and is therefore more subjective than the other forms of repetition. Clement Antrobus Harris [11] considers several additional categories of repetition that he calls *immediate*, *mediate*, *repetition with variation*, and *overlapping* or *interlacement repetition*. In immediate repetition an element is repeated immediately without any intervening matter, such as in the string of elements X X X X X X X. In mediate repetition, on the other hand, an element is repeated after other different elements have appeared, as for example in the string X a X a X. The *rondo* structure of repetition in music is one special case of mediate repetition. In the rondo the intervening matter changes constantly as in the sequence X a X b X c X d X e X f. Harris’ repetition with variation is the same as Brown’s intellectual repetition. By overlapping or interlacement repetition Harris means that a repeating pattern is formed by the superimposition of translated or rotated *identical* copies of an original pattern. Toussaint and Liu [6] proposed another type of repetition they call *emergent*, which is an extension of Harris’ overlapping repetition. Emergent repetition occurs if the repetition comes about by superimposing patterns that are *different* (non-congruent).

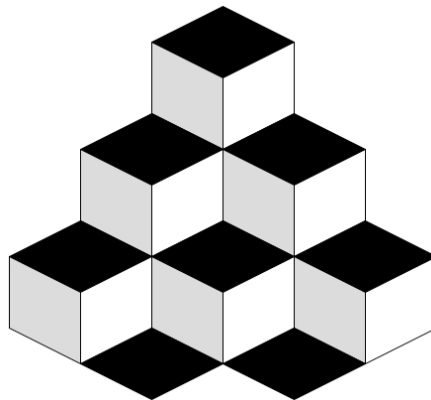


Figure 1: Reversal and bistable perception of the Necker cubes.

Optical illusions, subjective contours, “impossible” figures, pictorial ambiguity, multistable perception, and figure-ground reversals have also been extensively used in the visual as well as auditory arts, most compellingly perhaps by M. C. Escher [14]-[21]. A typical example of bistable perception discussed in [14] and [21] may be experienced with the Necker cubes pictured in Figure 1. Here the central corners of the cubes are sometimes viewed as sticking out towards the viewer, and sometimes into the page away from the viewer.

In this paper the most interesting and unique geometric frieze patterns found on the marble pavement of the Cathedral of Siena are analyzed in terms of a typology of repetitions, and from the point of view of the optical effects, such as the compelling multistable perceptions, that some of them engender in the viewer. Space limitations do not permit the analysis of all the friezes present in the pavement.

The Geometric Frieze Patterns on the Pavement

The first panel one encounters as one enters the cathedral through the main central front door portrays Hermes Trismegistus [1]. It is the only panel of the pavement that is framed by friezes that are heavily influenced by the ancient Greek and Roman meander curves [6]-[8], [23]. One of these friezes, pictured in Figure 2, is a unique variant of the ubiquitous Greek key pattern shown in Figure 3 (top). This frieze is not listed in the border design catalogues of Day [22] and Fenn [24] that list all the designs that they knew existed in Greek and Roman mosaics, as well as their own designs. However, in the combinatorial exhaustive computer-generated list available on the Internet web page called Greek Keys [25] it receives the name *edge-to-edge inverse meander with two turns*. Figure 3 shows how this frieze may be obtained from the classic Greek key. This type of repetition thus belongs to the *formal* type of repetition within the frieze itself but to the *intellectual* category with respect to different periods in time.



Figure 2: *Original variant of the standard Greek key meander pattern.*



Figure 3: *Construction of the frieze from the Greek key pattern of Figure 2.*

The frieze pattern immediately surrounding the Hermes Trismegistus panel shown in Figure 4 (left) is typical of the swastika meander patterns found on ancient Greek and Roman mosaics [22]-[24]. This illustration, taken from the book by Bruno Santi [1], rather surprisingly, crops off the sides of the frieze.

The completed design is shown on the right. Analysis of the main pattern reveals that it consists of two closed meander curves pictured in Figure 5. Furthermore, one of these is an identical copy of the other rotated by 90 degrees. Thus the types of repetition that characterize this frieze are formal across time, and formal as well as intellectual, literary, and interlacement repetition within the frieze itself. The repetitions present in the frieze are formal because the swastika patterns are repeated exactly, intellectual because the flowers in between the swastikas are repeated with slight variations, literary because the swastika has great symbolic meaning [13], and interlacement because the swastikas are created by two overlapping identical curves.

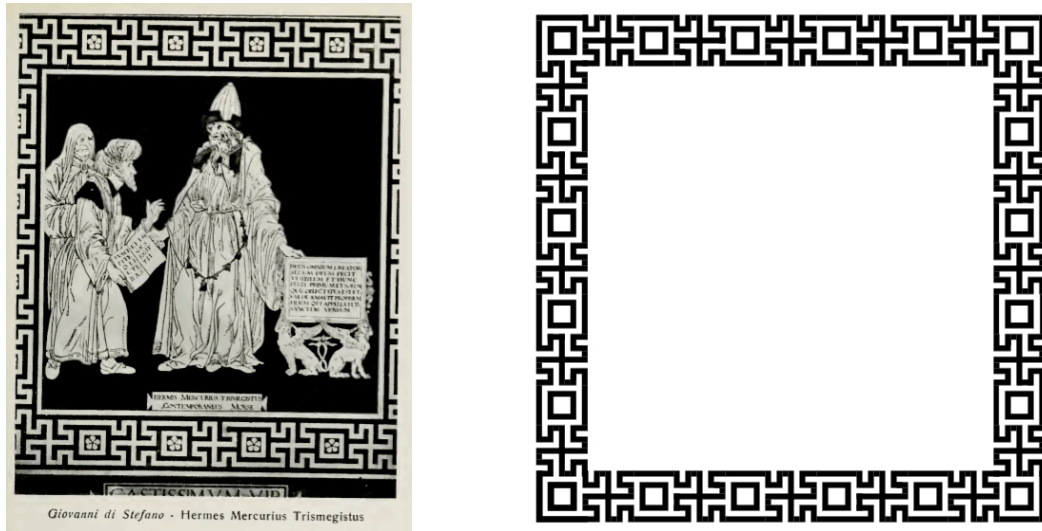


Figure 4: *The illustration from [1] (left) and a drawing of the complete meander pattern (right).*

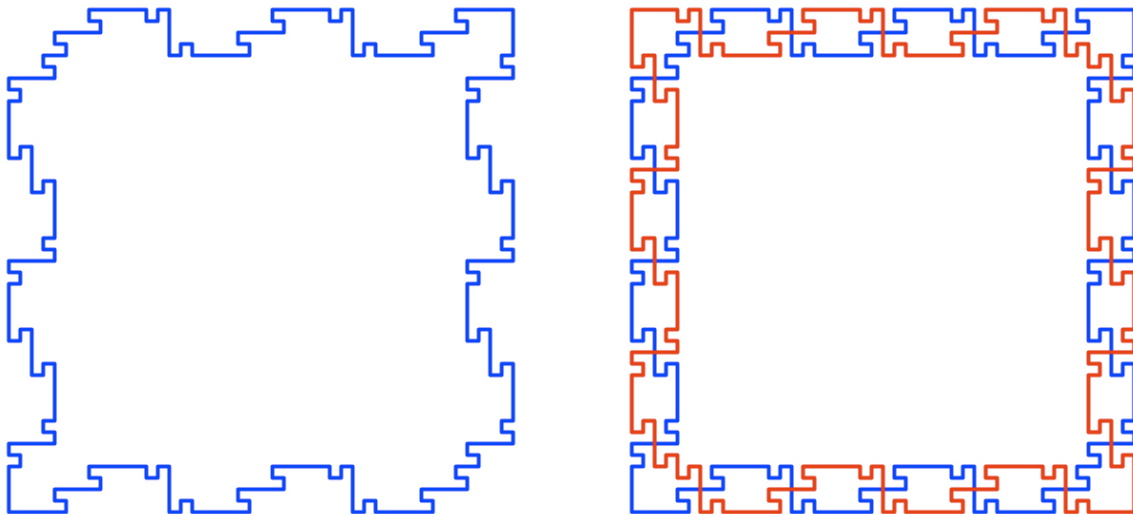


Figure 5: *One of the two meander curves (left), and both together (right).*

There are several frieze patterns that produce stunning 3-dimensional effects such as multistable perception. Therefore these friezes deserve to be assigned to the category of triumphant repetition [10]. One such example is the freeze surrounding the Wheel of Fortune shown in Figure 6. It consists of an

array of squares divided into differently colored pieces in such a way that as one gazes at the frieze a variety of different unstable structures are perceived. Staring at the individual squares produces a 3-dimensional square volcano-like “mountain” containing a square “crater” with shadows produced by light coming from a northwesterly direction. Staring at the cross points where four squares meet produces crosses that spontaneously switch from convex (mountains) to concave (valleys). Viewing the entire pattern more globally one sees a pattern of diagonal strips running from the lower left to the upper right.

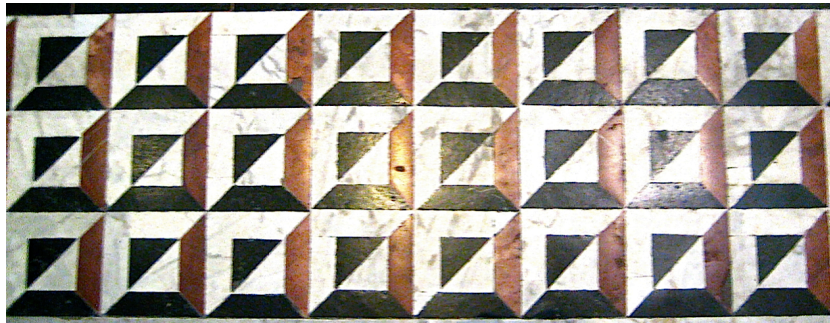


Figure 6: *The geometric frieze surrounding the Wheel of Fortune.*

Scattered throughout the pavement there are several friezes referred to as *step* friezes, made up of cubes. However, these cubes are colored in such a way as to produce a variety of multistable perceptions. One of these step friezes consisting of triangulated cubes is shown in Figure 7. Sometimes one perceives the 3-dimensional cubes as protruding towards the viewer. At other times one sees a collection of triangular pyramids viewed from the top. Furthermore, due to the choice of dark and light coloring of the triangles one may also perceive 6-pointed 3-dimensional stars as if illuminated by light coming from the north or the south, depending on whether the mountains making up the stars are convex or concave.



Figure 7: *Step frieze consisting of triangulated cubes.*

Another simpler but more compelling step frieze surrounding the Wheel of Fortune is shown in Figure 8. It consists of an unusual coloring of two layers of the simple chevron pattern shown on the right. The pattern on the right is bistable: sometimes it resembles the roofs of row-houses where the edges between black and white regions are seen as horizontal, and this perception alternates with that of Chinese folding

room dividers where the edges between black and white panels appear to be vertical. Because when putting together two copies of this chevron pattern on the frieze (left) the white and black colors are reversed, the figure has three non stable perceptions: the roofs of long row houses, the walls of tall room dividers, or a row of complete little houses with one chevron layer making up the roofs, and the other making up rather unusual concave corner walls of the houses.

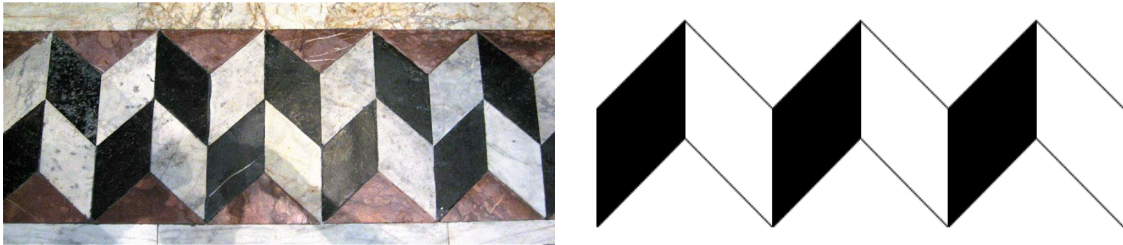


Figure 8: *Chevron-style frieze surrounding the Wheel of Fortune (left) and one bistable layer (right).*

The step frieze shown in Figure 9 (left) is not only multistable, but produces the perception of impossible figures [14] as well. It consists of pairs of cubes such as those shown in the diagram in the middle, separated by straight-line hourglass figures alternating with diamond figures. It may be obtained simply by cutting a diagonal strip from the type of step pattern shown in Figure 1, as illustrated in Figure 9 (right). This frieze has four unstable possible perceptions: a black-brown-black step with convex corner on the left or convex corner on the right, and a white-brown-white step with convex corner either on the left or the right. Moreover, this figure may also be perceived with both central corners of the cubes as convex (protruding towards the viewer). In this case the perception alternates between possible and an impossible figures. The impossible figure results from viewing the brown region (left) or white region (center) as a planar face, which cannot be the case in the real world when the central corners of the cubes protrude towards the viewer, since then the white diamond would have to be a “crevasse” flanked by two triangular faces, and there would have to be included a diagonal line partitioning the diamond into the two triangles. Two possible figures result when we perceive this white region as a hole instead, and the rest of the figure not as solid cubes, but rather as bands of paper which are folded at 90 degrees along the grey-black edges such that they are either both convex or both concave with respect to the viewer.

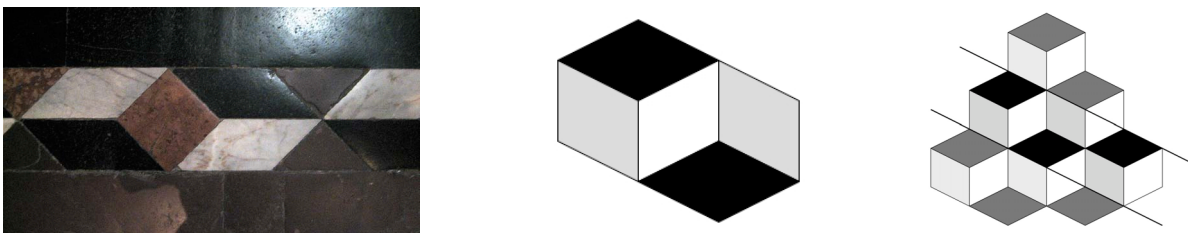


Figure 9: *Multistable step frieze including the perception of impossible figures.*

Two additional frieze patterns in the cathedral pavement are worth mentioning due to their unique complexity and originality of design. They look like 3-dimensional space stations, for lack of a better term. One is the *Naspatio* frieze [2] at the bottom of the panel titled *Judith and the Victory at Bethulia*, and the other is the border between *Judith and the Victory at Bethulia* and the *Massacre of the Innocents*. Both are pictured in Figure 10, and clearly belong to the category of triumphant formal repetition by virtue of their tremendous emotional response they elicit in the viewer, particularly from the geometric point of view.



Figure 10: *Complex three-dimensional architectural structure frieze patterns.*

Finally we close this paper with an example of a fascinating complex frieze pattern found on the marble pavement on the outside of the Cathedral facing the main front entrance. This frieze, shown in Figure 11, is most unusual in that it is completely different from all the friezes inside the Cathedral. It appears to consist of a long thin branch of a tree with twigs cut off (perhaps thorns?) around which a ribbon is wrapped in a helical fashion. This type of repetition surely also deserves to be in the category of triumphant repetition by virtue of its uniqueness and originality.

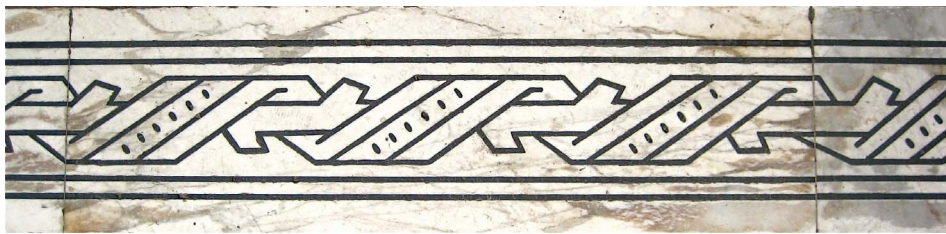


Figure 11: *Complex frieze pattern on the marble pavement outside the Cathedral entrance.*

Acknowledgments

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