Using Mathematics in Art

Michael Field
Department of Mathematics
University of Houston
Houston, TX 77204-3008, USA
E-mail: mikefield@gmail.com

Abstract

In this article we discuss some of the issues concerning the possible relationships between art and mathematics. In particular, we address the questions of whether or not mathematics *is* art – or at least contains a significant artistic component – and how one might use mathematics and mathematical artifacts *in* art. We illustrate with a number of examples taken from our own work (the examples appear in color on the CD-ROM).

folly for to need to seem to glimpse what where - where - what is the word[1]

Introduction: Art & Mathematics

For several years past I have been intending to write an article on my thoughts and experiences about using mathematics in art. However, as a professional mathematician, working in the geometric theory of dynamical systems, putting pen to paper has turned out to be harder than I anticipated. In part, this is because the style of mathematical exposition is so very different (easier I could say) from that expected in the humanities[2]. Thus, both my conclusions and hypotheses are necessarily (and deliberately) rather ill-defined and imprecise – and this situation can often lead to strong opinions being held all around. An instance of life's uncertainty principle: "The strength of opinions on an issue is inversely proportional to the available information". In this article, I have tried to express some of my concerns and aims and then relate some of my experiences about how I have approached the use of mathematical ideas in creating graphic art. I have not attempted detailed historical cross-referencing or even referred to other contemporary mathematically influenced art; that would have made the article far too long and formal. As well, I am not an expert on art or the history of art. But I do want to encourage further conversation and thought about the issues I raise as well as challenge some widely accepted beliefs.

A little background: in 1988, following a visit to Houston, I started developing programs to visualize symmetric chaotic attractors. Using these programs, Marty Golubitsky and I wrote a book in 1992, *Symmetry in Chaos* [1], which was designed not only to show some pretty pictures of symmetric chaotic attractors and fractals but also explain (and propagandize[3]) the mathematics underlying the images. Later, after I moved to the University of Houston, I taught several interdisciplinary courses to junior and senior students in the Art department. These courses were mainly about symmetry and its representations in art and design. A significant amount of class time was spent using my computer programs as an aid to the understanding of symmetry. I have also led seminars for Houston area teachers in the *Houston Teachers Institute* (1998-2009 – now discontinued) and used the programs in two of those seminars (some of the images created by students and teachers can be found at http://www.math.uh.edu/~mike/PATTERNS).

1 From *What is the Word*, 1998, by Samuel Beckett.
2 For a inspiring example of how one might write about mathematics, see the recent book by Borovik [1].
3 *Propaganda:* in the current climate of hostility towards mathematics and science it can be helpful to have some striking visual examples that illustrate what mathematics is about in a non-threatening way.

At the outset, I wondered about the artistic potential that might lie in these images of chaotic attractors and fractals. Certainly, with their intricate structure and fine detail, I felt comfortable viewing some of them as a little akin to fine art – mathematical Fabergé eggs if you will. While I did not feel the images were in any sense 'art', I did start to think, over many years, about how one could use mathematics in art – that is for artistic purposes (as opposed to illumination of the mathematics). Of course, part of this was pondering whether or not the images of symmetric chaos could be consciously used in a more artistic way[4]. One thing I noticed from the classes I taught and seminars I led, was that the response to the images and the way people chose to color them was highly individual (and I felt quite culturally based). This seemed a promising sign. Even better, it was possible to create quite ugly images with the tools at hand.

What, if anything, is the relationship between art and mathematics? My observation is that artists are often unreceptive to the idea that mathematics has much to contribute to art (early on, I was often warned that artists would not respond well to mathematical art work – though personally I have never encountered any such hostility). On the other hand, many mathematicians claim that art and mathematics are intertwined and that mathematics even provides a bridge between art and science. This association is often based on the significance of beauty and pattern in mathematics and was perhaps most famously expressed by G H Hardy in *A mathematician's apology [3]*

> *A mathematician, like a painter or poet, is a maker of patterns.*

Hardy goes on to say that, "If his patterns are more permanent than theirs, it is because they are made with *ideas.* A painter makes patterns with shapes and colors, a poet with words. A painting may embody an 'idea' but the idea is usually commonplace and unimportant." For Hardy to make this argument, he has to claim that mathematics is essentially a creative process rather than one of discovery (that is, of pre-existing results/ideas). Otherwise, the comparison would have been better made with a miner of precious stones or an archaeologist. and then it would have been harder to make the implied claim about intellectual superiority. Whatever else, I think Hardy's quote helps to explain the distrust some artists have about the intrusion of mathematicians into the art world: "not only is my picture more beautiful than yours but it is a reflection of a deep universality that is quite beyond your limited comprehension". Nevertheless, artists do sometimes adopt mathematical ideas or terms as a way of adding depth or significance to their work; the classical example being that of the *golden mean:* mystic numerology and geometry is certainly a feature of some art. However, in spite of the numerology embedded in, for example, the music of Bach and other composers, I am not convinced that it is an important attribute of the music (there is no issue that mathematics comes into the description of music). Similarly, while the golden mean certainly appears to have influenced some architects and designers, I am far from convinced that it plays a major role in our perception of harmony and beauty.

For a contemporary mathematician's viewpoint, there is a short article "Art and Mathematics" by Field's Medalist Michael Atiyah, which appeared in a recent focus issue of the *Notices of the American Mathematical Society* [4] on "Mathematics and the Arts". Atiyah stresses the importance of beauty in mathematics and argues that architecture is the most appropriate of the arts to compare with mathematics[5]. While I cannot agree with the syllogism that 'art is beautiful, good mathematics is beautiful and therefore some mathematics is art', I do feel that the comparison with architecture is a good one. Architecture is (perhaps was) a *collaborative* effort, so is mathematics. Consider, for example, the massive investment by European society building cathedrals and churches. Some of these buildings literally took hundreds of years to complete. The construction involved artisans and craftsmen at every level of the construction. None of these edifices was perfect at every scale – one can usually find the good, the bad and sometimes the ugly. All this is very much like contemporary mathematics. We want to

4 I feel that art is *created*, not discovered. That affects the way I respond to mathematical 'art' imagery.

5 For another viewpoint, see the book 'Symmetry' by the famous mathematician and physicist Herman Weyl [5].

focus on the best but the reality is a mix of startling insights and beauty with much that is mundane and often positively ugly. The difficulty I feel is with the word *beauty*. I do agree that beauty, in the sense of elegance of argument and proof, is very important in mathematics. Also important is the idea of insight; crucial for mathematics is the idea of truth – in the sense of the *right answer*. I find it much harder to quantify what is meant by beauty or truth in art. There is a trivial sense: the pretty picture or the beauty of the English language in a Shakespeare sonnet or play. This seems to be what Hardy is picking up and then dismissing as commonplace. Perhaps Hardy is partly correct in his comparison but this is at the expense of missing the whole point and meaning of art. Of course, if one defines merit and worth in terms of elegance, beauty and truth, then inevitably mathematics emerges as a superior art form. This argument and reasoning is not likely to elicit a positive response from the art community; nor is it, in my opinion, a satisfying or accurate description of art. Indeed, I do not see a convincing argument that beauty or pattern is *the essential feature* of great art. Often in literature, for example, the power of the work comes from the use of metaphor and analogy. Necessarily, ambivalence, associations and resonances in the language play a key role (rather than a beautiful pattern or design). I claim this must be so since the province of much (great) literature and art is dealing with questions of the human condition and the nature of existence; these questions never admit simple yes/no answers and indeed, in this domain, there are no simple truths (even though philosophers and theologians sometimes claim the contrary – often to everyone's eventual detriment). Take a play, such as *Hamlet* – the meaning of which still provides work for literary critics and students of English – or a novel like *Crime and Punishment.* Are beauty or pattern the concepts that spring first to mind when thinking about these works? This is not to downplay the importance of the language of Shakespeare – but it is being used for very particular effects and it is those effects we remember. On the question of language, let me give a simple example of what I mean by 'resonance'. The following quotation is taken from the novel *Murphy* [6] by Samuel Beckett.

> w*ho knows what the ostrich sees in the sand?*

The ambiguity of this question (or is it a statement?) is the antithesis of 'good' mathematical writing[6]. The marvelous character of the sentence is that it cannot be tied down to just one meaning – the literary equivalence of the Necker cube. It is also reminiscent of Philip Guston's observation: *Look at any painting. It's like a gong sounding; it puts you in a state of reverberation.*

Viewed in this light, literature seems closer in spirit to science than to mathematics. Science seeks truth but has to assume it is working with an approximation to some imagined but unattainable truth (even unknowable – as in quantum mechanics). Literature also seeks truth that can only be described in ambiguous and metaphorical terms. Attempts to describe human 'truths' in absolutely precise terms typically lead to either banality or obscurity (often both). These issues are far too complex to break down into simple logical or mathematical statements. I feel strongly that the attempts to quantify the social sciences, philosophy and even religion over the past 100 years have often weakened rather than strengthened these fields. Worth should not necessarily be evaluated by the applicability of scientific method and the potential for producing numeral data.

I believe many of the previous arguments apply to two-dimensional art. Is, for example, the art of Rembrandt or Francis Bacon beautiful and is that the most *important* quality of their art? The question here is not about the artist's technique or the prettiness or otherwise of the canvas: the Dordogne is a spectacularly beautiful area of France, but it is not art. Rather, the question is about the artist's perceptions (and the quality of those perceptions) and what the artist is indirectly saying about us – humankind. Although much twentieth century art has had a strong geometric flavor – especially in sculpture but also in painting – when I look at great twentieth century paintings like *Guernica*, beauty is not what springs to

6 In this case the effect was amplified by the 'answer' – *A can of worms*, made by a Chinese system administrator in Houston!

my mind. Conversely, although Escher figures strongly in the Math-Art world, he has not received nearly so much positive attention in the art world. As a mathematician, I personally find Escher's work interesting because of the patterns and illusions but, although this may seem heretical, I do not particularly enjoy his work as art.

Using Mathematics in Art

In the remainder of this article I want to discuss how one can use mathematically generated objects as part of a toolkit for creating (planar) art. I do not mean by this the attractive coloring and presentation of an interesting mathematical object – for example a ray traced surface or Julia set. Rather I am interested in the potential for expressiveness, ambiguity and artistic interest that may lie in mathematical artifacts. There is nothing very novel about this when it comes to sculpture. The shape and geometry of modern abstract sculpture carries its own sensuality and harmonies; the effective use of materials and textures make it possible for the sculptor to achieve striking effects that go far beyond a simple representation of a Mobius band or other particular mathematical object. In summary, my interest is not in providing an instructive and attractive visualization of a particular mathematical object – though that can certainly be a worthy task – it is in asking how far the object might be used for artistic goals. The approach I adopt here will be to describe some of my experiences over the past twenty years working with chaotic attractors and fractals and the way I have approached some of the issues about expression. I illustrate with a few examples (these appear in color on the CD-ROM). I should add (unapologetically) that I do not discuss the *meaning* of the images or exactly what I am trying to convey in some of my work. As I am sure most artists and writers would agree, less is sometimes more.

The Materials and Design

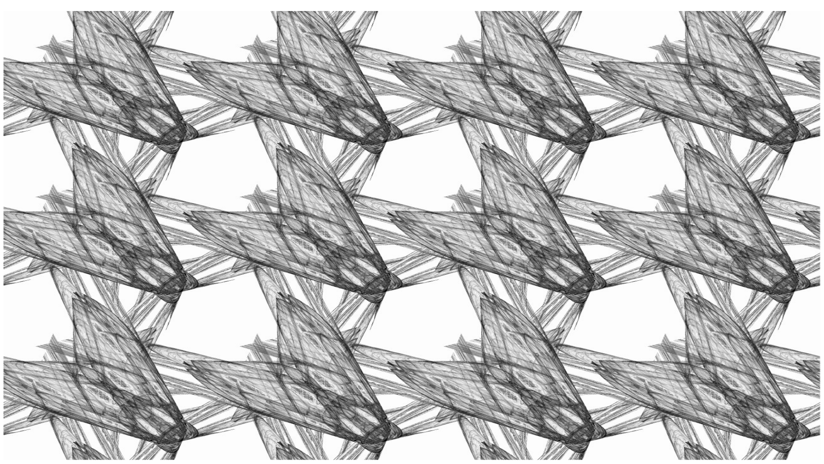

Figure 1: *Fly Quilt*

I use algorithms that generate either symmetric deterministic attractors or symmetric fractals. These may be represented as either 'bounded' objects or planar repeating patterns. I often introduce talks with Figure 1 – *Fly Quilt.* The pattern is a planar repeating pattern of type **pm**[7] and was constructed using an iterated function system defined on the two-dimensional torus. The pattern on the torus was lifted to the plane where it appears as a repeating pattern. The image is colored in a grey-scale. The first element is the design. Originally, I was experimenting with an algorithm that gave an image that suggested *Fly Quilt*. The program I have developed allows variation of the pattern in an interactive way until one gets the desired effect. For *FlyQuilt*, the symmetry was crucial – it suggests motion (and for me is reminiscent of a barbecue in outback Australia). In summary: algorithms, symmetry effects, and intent.

7 For repeating pattern notation, see the book *Symmetries of Culture* [7].

Fly Quilt was constructed using an iterated function system. Deterministic mappings typically lead to quite different effects and textures. In Figures 2 and 3 we show two examples. *Thorns*, a bounded pattern with 5-fold rotational symmetry, was perhaps my first attempt to consciously create an artistic effect (but I will not go into the context). *Endgame* has all that ambivalence and association that I like – it is part of a repeating pattern of type **pgg.**

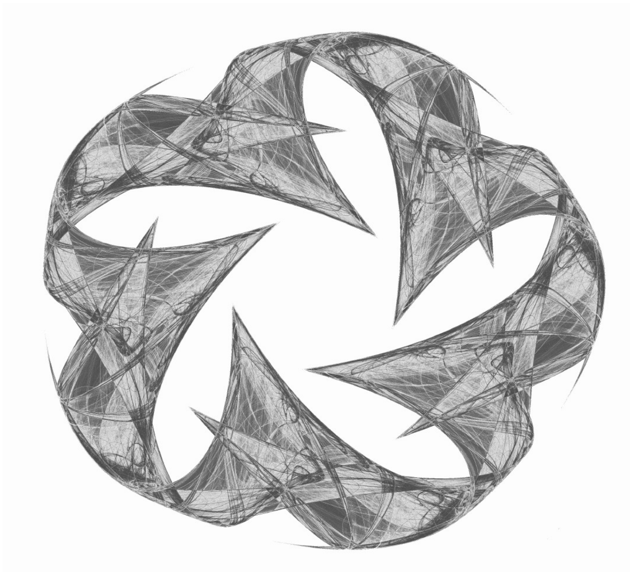

Figure 2: *Thorns*

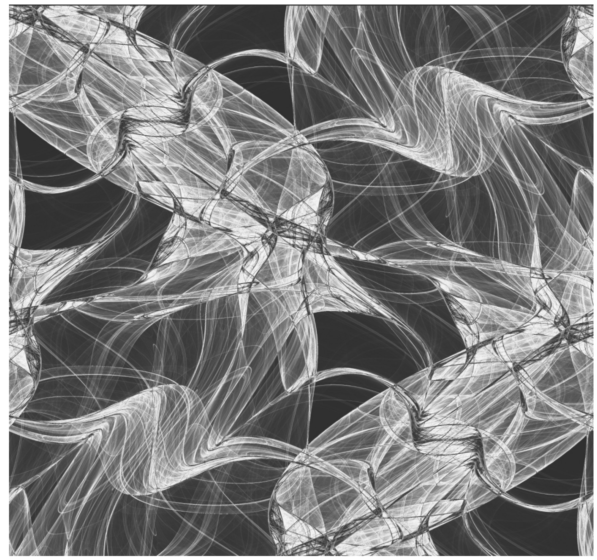

Figure 3: *EndGame*

Much of the time I have spent working on these programs has been on designing new *algorithms*. As part of the creative process, I attempt to design an algorithm (really a family of algorithms) that give me the effects I want. In Figures 4 and 5, I show some simple examples of algorithms that lead to geometric angular designs (these are based on an iterated function system on the 2-torus).

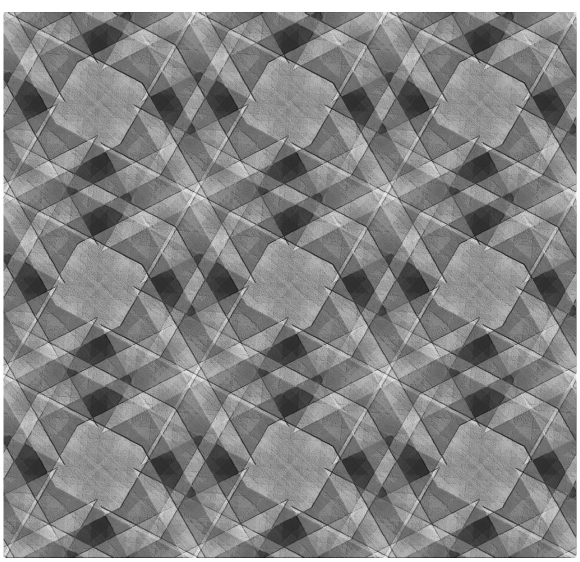

Figure 4: Abstract design #1

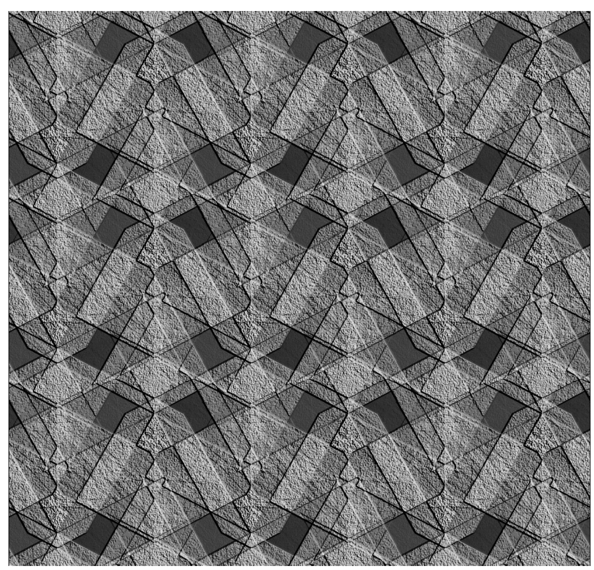

Figure 5: Abstract design #2

(The first repeating pattern has **p4** symmetry, the second is of type **pmg**.)

I find one of the most interesting and challenging parts of the creative process to be the use of color. 'Two-color' repeating patterns – half the symmetries preserve color, half the symmetries interchange color – have great artistic potential. Figures 6 and 7 show two examples (both constructed using the *same* iterated function system). Note the depth in the first image and the holographic effect in the second. Two color designs have enormous potential for incorporating illusion and ambiguity.

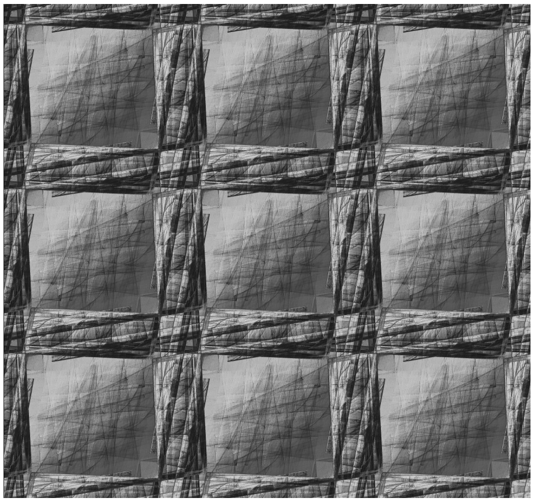

Figure 6: *HellFire III*

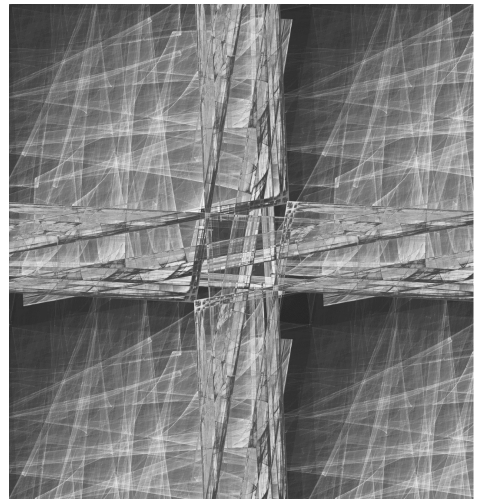

Figure 7: *EnduringIllusions*

Each symmetry carries its own 'dynamics' and generates its particular psychophysical effects on the viewer. In *Thorns*, for example, a sense of motion is conveyed by the use of rotational but not reflectional symmetry. In Figure 8, an example is given of the static design that can result if one uses a pattern with rectangular symmetry **pmm** (many reflections) On the other hand, Figure 9 shows a repeating pattern with symmetry **pgg** – no refection symmetries at all (but many rotational and glide-reflection symmetries). All of this becomes much more interesting when one works with two-color symmetries. Often, using color, one can play a static symmetry, such as **p4m**, against a dynamic symmetry like **p4**.

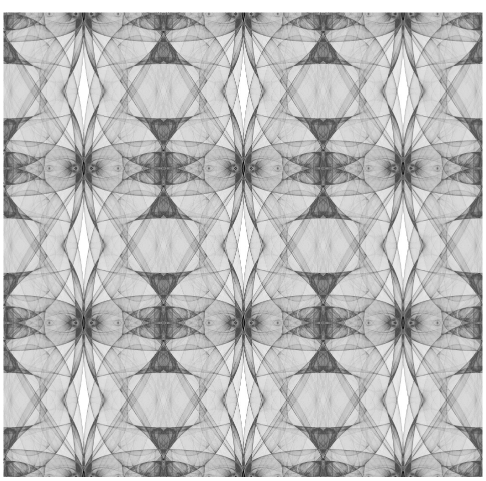

Figure 8: *StudyForAlhambra*

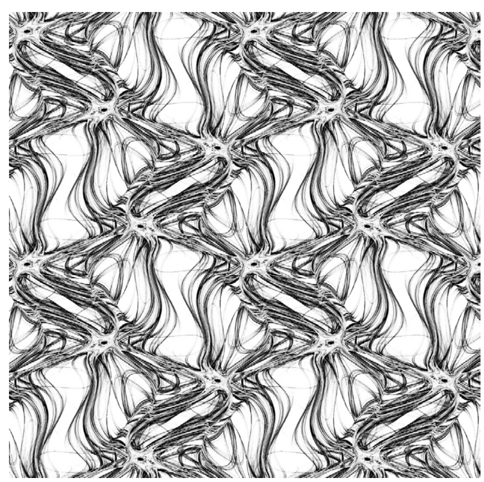

Figure 9: *NeuralNet Quilt*

Even though some symmetries can have very dynamic effects, restricting to symmetric patterns is too much of a constraint. Figure 10 shows an example of a composite – a mix of the Sierpiński triangle and 'something' with 11-fold symmetry. I won't attempt to describe the coloring algorithms here – suffice it to say that they are complex and take account of both dynamics and symmetry.

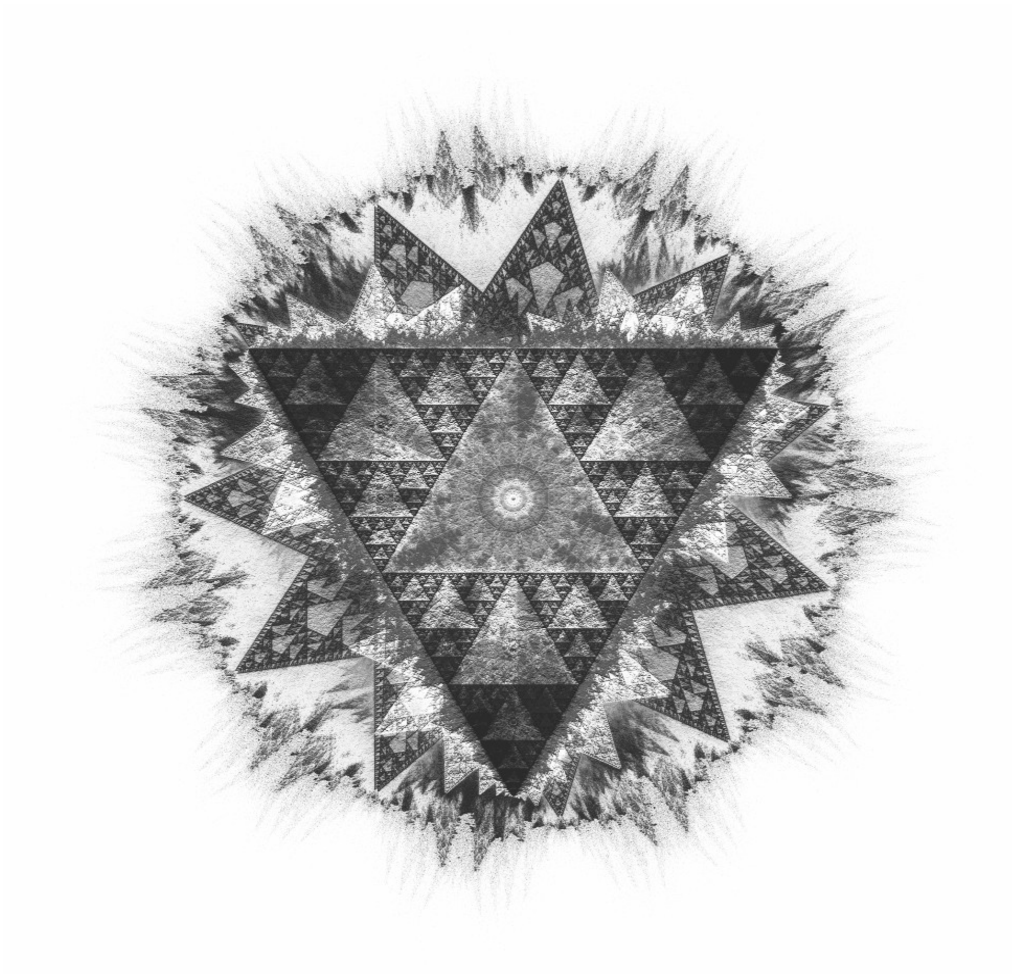

Figure 10: ButIsItArt

Finally, a more recent image – Figure 11 – part of continuing experiments to see what is achievable using ideas based on chaotic dynamical systems.

Conclusions and Summary

We all know that Art is not truth. Art is a lie that makes us realize the truth, at least the truth that is given to us to understand, Picasso.

I challenge the assumption that because parts of mathematics are beautiful then it follows that attractive visualizations of mathematical objects must have an artistic component. In this article, there is no attempt to illustrate the beauty of mathematics or to claim that because it is mathematics it must be art. Rather, I am trying to use mathematical technology to achieve certain desired (non-mathematical) ends. While I do not believe that mathematics of itself adds weight or profundity to art, I do believe that it can be used for expressive and artistic ends.

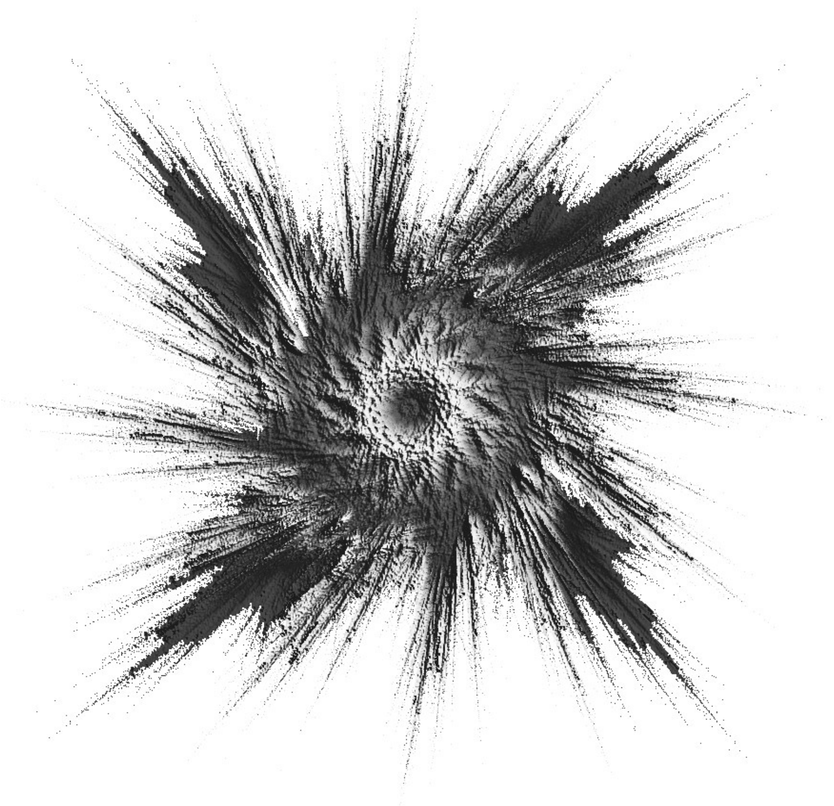

Figure11: EUFractal

References

Doubt grows with knowledge, Goethe

[1] A V Borovik. *Mathematics under the Microscope* (AMS publication, 2009).
[2] M J Field and M Golubitsky, *Symmetry in Chaos* (OUP, 1992); 2nd revised Edition, (SIAM, 2009).
[3] G H Hardy, *A mathematician's apology*, Cambridge University Press, Canto Edition, 2001, with a forward by C P Snow, 1967.
[4] M Atiyah, 'The Art of Mathematics', *Notices of the AMS*, (57) (1) (2010), 8.
[5] H Weyl, *Symmetry*, Princeton Science Library, 1983,
[6] S Beckett, *Murphy*, (page 122, chapter 9), Calder & Boyars Ltd, London, 1970.
[7] D K Washburn and D W Crowe, *Symmetries of Culture,* University of Washington Press, 1998.