Borromean Rings & Three Fold Knots

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Abstract

Since 2007, I have been making textiles based on Borromean Rings and 3-part knots - 3 corners or links - trinity, triqueta, or trefoil structures. This paper covers making 2D images into 3D arrangements, ratios and structures.

Making Maths Accessible Through Art

In 2007 I had a second solo exhibition on Lindisfarne. 'Holy Island' is one of the centres of early Celtic Christianity in the UK, so I wanted to make some new work, which made reference to celtic knot work. In 2004 I had made 4 floaty banners from transparent fabrics (sheers), with celtic knot motifs from the Matthew Lindisfarne Gospel page [1], trapped between. A visitor mentioned her friend was working on Borromean Rings, which I had never heard of before. In 2007, I decided to look at 'threesome' knots and ring structures. I made two large wallhangings, several small ones and half a dozen 3D versions of the structures. I also continued to use the Fibonacci sequence with some interesting results.

Mission - visualising 2D shapes in 3D Many ordinary people, especially women are scared by the word and concept "maths", even though they do maths everyday intuitively. I learnt modern maths in a comprehensive school, which for me was about pattern making, but I lost the plot just after quadratic equations and didn't rediscover the magic of numbers until I was an adult.

When I teach students about the Fibonacci sequence I often see their faces glaze over, until I get them to do a practical collage and they see the pattern coming together. When I talk about my work to textile groups, it obviously involves mathematical ideas which most of my audience have never considered, so part of my mission statement is to make maths accessible.

In studying the Celtic knots in the Lindisfarne gospels and books such as Aiden Meehan's [2] it suddenly occurred to me they were woven structures with just a few yarns. I trained as a weaver so I could visualize the structures ups and downs. Quilter's often make Celtic knot designs using bias binding tapes - a very laborious method. I wanted to push boundaries. I realized if I used Bondaweb (a commercial heated fabric glue) on the coloured fabrics I needed, and cut concentric circles out of them I had masses of circles to play around with in my designs. The trick was to hide the cut in each circle under one of the other rings crossing over it. Thus my wallhanging technique was sorted.

Next I wanted to help people see that 2D motifs in books were actually 3D objects, so I needed to make tubes. During my Art Foundation course (1991) Ann Sutton [3], a weaver gave a talk. While the diploma fashion students were totally unimpressed, I was fascinated. Not only was Ann working with interesting number/colour systems which led to my own Mathematical Magic colouring system, but she knitted rainbow coloured tubes and made large and interesting sculptures with them.

I didn't want to copy Sutton's work, so resolved to make my tubes in fabric and thread from a patchwork/embroiderer's perspective.

Borrovari (2007) or How Many Ways Can 3 Rings Be Arranged? I came across P R Cromwell's Borromean chart on the internet [4], his theory is that there are 64 possibilities for placing 3 circles to look like Borromean Rings (as a solid 2D motif) with 10 distinct variations caused by three processes:

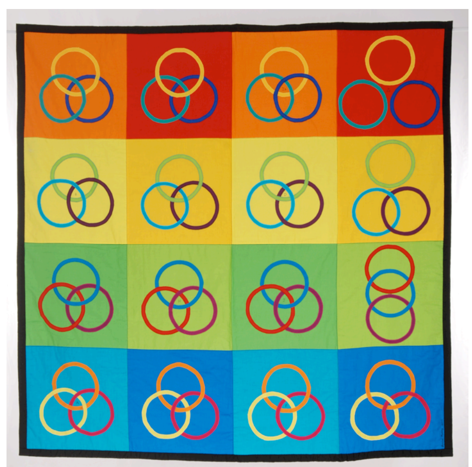
a) Rotation by 120 degrees b) Reflection
c) Reflection in the plane of the pattern, which means that all the crossings are switched.

Cromwell's chart shows broken rings in black to show the crossovers. The differences are not immediately obvious unless you look very carefully, so in order to help this make sense for non mathematicians, I rearranged Peter Cromwell's original table, so the patterns are rearranged into rows of: separate rings; two joined, one loose; three in a chain; or three completely intertwined.

I also chose 3 separate ring colours for each horizontal row, so each ring is consistently in the same position, so the varieties of crossings should be more obvious. For added interest, I alternated the the background block shades and also changed the ring colours for each row. I have quilted the piece minimally - by machine along the edges of the background blocks and by hand in the centre of each block, where either the code letter or the structure icon acts as a 'tying' stitch.

Peter Cromwell's table

Row 1	**Mathematicians' Borromean Rings**
	three rings joined to each other
a)	over, under, over, under
Row 2	**Hopf Link with a trivial knot**
b) c) d)	one separate ring & a 2-ring chain
Row 3	**(3,3)-Torus Link**
	all rings linked to one another
e)	over, over, under, under
	under, over, over, under
	over, under, under, over
f)	over, over, under, under (all rings)
Row 4	**3-Component Chain**
g) h) j)	three rings linked in a row
Row 5	**3-Component Trivial Link**
i)	3 separate rings laid on top of each other in different orders



My table – alternative layout

Row 1	
i) ia) 1b)	3 separate rings
Row 2	
b) c) d)	one separate ring & a 2-ring chain
Row 3	
g) h) j)	3-link chain
Row 4	
a) e) f)	Borromean / 3 rings linked to each other

Right hand column is the underlying key to the structures

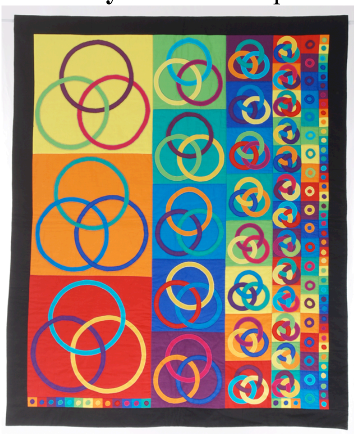
Figure 1: *Peter Cromwell's chart* **Figure 2**: *Borrovari (c54"/137cm square)*
Figure 3: *My version of Cromwell's chart*

Fiborro (2007) Borromean Rings in a Fibonacci Grid I am working towards a book on how to use the Fibonacci sequence in textiles and other related arts and am experimenting with different Fibonacci grid structures in my work.

For this quilt I worked out the optimum size using (**1**), **1**, **2**, **3**, **5**, **8**, and **13** width blocks, the first 6 numbers in the sequence, which is obtained by adding the previous 2 numbers together to create the successive number. It is a wonderful sequence with many magical properties, which often lead to other fascinating number related structures

Colour choices I chose 8 background colours. The sequence is red, orange, yellow, light green, emerald green, turquoise, royal blue and purple. I added 5 more to make 13 ring colours. The additional rings are pink, light pinky/purple, orangy yellow, dark purple and light turquoise

Grid layout For this quilt I chose to have 6 columns, which are, from left to right



3 blocks of **13**" units (39")
5 blocks of **8**" units (40")
8 blocks of **5**" units (40")
13 blocks of **3**" units (39")
20 blocks of **2**" units (40")
40 blocks of **1**" units (40")
The unit sizes decrease from left to right, while the number of units in each column increase according to the Fibonacci sequence until the 2" blocks, which is 1 short of the next Fibonacci number **21**. 1" blocks 'fill in' the gaps on the 39" columns - there are 16 extra 1" units - 56 in all – just 1 out from the sequence no **55**

Figure 4: *Fiborro 2007 (36 x 44"/91.5 x 122cm)*

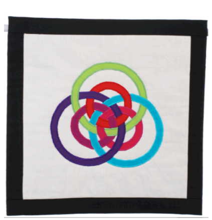
The backgrounds are laid in a vertical zigzag format
- up from red at bottom left, through orange to yellow on column one (13"units),
- down from green to purple on column two (8"),
- up from red to purple on column 3 (5"),
- down from red to purple an on again to dark green on column 4 (3"),
- up from turquoise on column 5 (2") and
- down again on column 6 (1") through several colour cycles

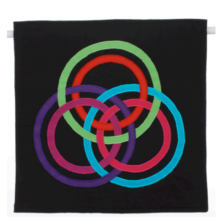
The background colour sequence continues along the bottom, from right to left, in the gaps beneath the 39" columns, which are the odd numbers in the Fibonacci sequence (apart from 5). Unfortunately the colours did not follow through completely - red and orange repeat themselves at the 'join', it would have been a wonderful mathematical coincidence if they had followed through completely, but I'm sure most people won't notice this at all!

Ring colour placement are fairly random on the first 4 columns, I simply chose the best three colours for each background, however on columns 5 and 6, the ring colours proceed in an 'extended rainbow sequence', where the background colour would be the same as the ring, I swapped the ring with it's successor and then continued the sequence after the swap

Ring sizes Each Fibonacci size ring sits on the next larger size background unit: so 13" units have 8" rings, 8" blocks have 5" rings etc. Since all the rings are ½" thick throughout, an exciting but unexpected effect happens as the spaces between the rings decrease from column 1 to 6 and the Borromean rings get closer together creating different optical effects on the shape. Column 5 has 1" circles on it, from which I ultimately decided to cut out ½" circles for column 6 (1")

The Relative Sizes of Rings as in Fiborro, I discovered when small diameter rings are used together the interlacing at the centre is very close. If you use larger rings, the spaces can be more even. This is particularly obvious on double Borromean structures





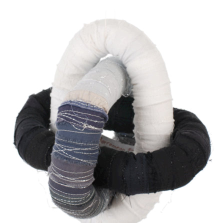


Figure 5 *Double Borromean, Closed Links (white) 13" 33cm* **Figure 7** *Borromean Rings 2 (BWG) 12"*
Figure 6 *Double Borromean, Open Link (black) 30cm (all 2007)* **Figure 8** *Borromean Rings 3(PGB)*

3D Borromean Ring & Knot Sculptures These are 3D renderings of line drawings**.** Some ring proportions work better than other. In Borromean Rings 4 the links are too long for their diameter. In Trinity/Trefoil, the tube is too short to show off the structure well. I didn't expect the structure to become a pyramid though. I have found a 1:4 ratio tube works quiet well apart from complex knots.

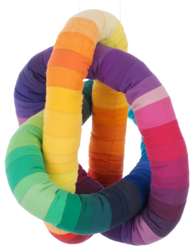
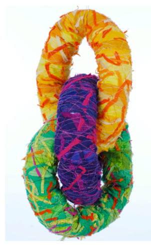
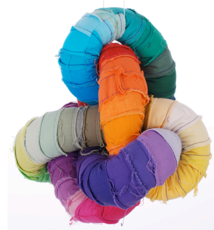





Figure 9 *Borromean Rings 4 (Rainbow)* **Figure 10** *Chain Link 1* **Figure 11** *Trinity/Trefoil* **(***all 2007)*
Figure 12 *Chain Reaction - the Rainbow Nations' Olympiad* **(***2010) 24" x 6½" LM [5]*

Chain Reaction - the Rainbow Nations' Olympiad (2010) Preparations for the 2012 Olympics are in the news a lot currently in the UK. During my 2009 shows many people thought my small wallhangings looked like the Olympic logo. The logo & it's colour arrangement is highly protected, so I made a piece based on it's structure but extending it to the seven colours of the rainbow: The original logo was designed in 1913 & first used in 1920, so you could say this is a 90^{th} anniversary tribute. The rings originally represented the 5 continents won over to the Olympics - America, Europe, Asia, Africa, Australasia (now Africa, America, Asia, Europe, Oceania) but if you included the 2 America's - North & South & Antarctica you would have 7 rings.

The logo colours were taken from the national flags then in use, and aren't arranged in a sequential scheme, whereas my piece has the colours in a ovalish circuit: Red, Orange, Yellow, Green L-R on the top, & Blue, Indigo (purple), Violet (reddish purple) R-L on the bottom row. The structure can be hung in various ways, the surprising thing is when you make it in reality, it is a simple chain not a totally interlocking shape! Most people will see it as interlocking unit! When this piece is hung correctly, all the top row rings face right and the bottom row faces left, effectively part of a to chain-mail structure, a third row would repeat the first and make it start to interlock.

Final thoughts

I thoroughly enjoyed discovering the mathematical patterns behind these structures. I was very pleased with the unexpected optical effects changes of scale created. I am currently making some huge Borromeans and planning some smaller scale chain-mail versions - an exploration for years to come.

References

[1] Lindisfarne Gospels presumed Eadrith c715, some facsimile on Lindisfarne, modern facsimile at Durham Cathedral, original in British Library

[2] Aiden Meehan, *Celtic Knots, Mastering the Traditional Patterns, A step-by-step guide*, Thames & Hudson. 2003. 0-500-51126-8

[3] Diane Sheehan & Susan Tebby, *Ann Sutton*, Crafts Council/Lund Humphries. 2003. 0 85331 885 9

[4] P. R. Cromwell, Liverpool University http://www.liv.ac.uk/~spmr02/rings/types.html Mathematicians' Borromean Rings/ Hopf Ring with Trivial Knot/ 3,3 Torus Link/3-Component Chain/ 3-Compenent Trivial Link. The labels a-j in the table are chosen to agree with those in the article: P. R. Cromwell, E. Beltrami and M. Rampichini, `The Borromean Rings', *Mathematical Intelligencer* 20 no 1 (1998) pp53-62. Quoted by permission PRC.

[5] All photos JB = Janine Boyer, except LM = Louise Mabbs