Locally and Globally Regular Toroids with Less than 16 Hexagonal Faces

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Abstract

In 2008 [1], we presented three classes of equivelar toroids. We have shown examples of al combinatorial types of toroids for face number $F \leq 12$. The data which uniquely determine these polyhedra are described in [2]. As a continuation of [1], here we show further examples for all combinatorial types of toroids with $F \leq 15$. These constructions represent outstanding mathematical achievements, and they also have significance in the art as sculptures having pleasant visual appearance. Such sculptures will be shown at the exhibition of the conference.

A polyhedron is called a *toroid*, if it is a torus in topological sense. A toroid is *locally regular*, if the number of edges meeting at each vertex is the same, and all of its faces are polygons with equal number of sides. A polyhedron satisfying these two conditions is usually called now an *equivelar* polyhedron (Brehm and Wills, 1993); here we use the term *locally regular* in the particular case of toroidal polyhedra. It is a requirement that the polygons must not be self-intersecting, and the polyhedron must not have overarching faces, i.e. two faces must not have common vertices more than two, and if they have two common vertices, then they have common edges as well.

A toroid is called *globally regular* (or, simply, *regular*), if the group of its combinatorial automorphisms is transitive on the flags. (A *flag* is a triple consisting of a vertex, an edge and a face, which are mutually incident). The F9 B and F12 D are globally regular toroids.

We attach to our paper on the conference CD the maps of all the locally regular toroids with face number $F \leq 15$; we give necessary condition for the global regularity of these maps; finally, we give a one or more polyhedral realizations of these maps. These realization are given via the software Euler3D [3]; the corresponding files contain also their numerical data.

Here in the Appendix we present some drawings of maps belonging to toroids with hexagonal faces, for $13 \leq F \leq 15$.

Finally, we remark that these polyhedra were obtained, using duality, from the polyhedra of type {3,6}, which can easily be constructed by the method described in [1,2]. Constructing polyhedra of type {3,6} with larger and larger number of faces is more and more easier. The contrary is true in the case of their duals, i.e. polyhedra of type {6,3}. Constructing the latter is more and more difficult, and they are less and less interesting mathematically. On the other hand, from aesthetic – artistic - point of view they may still be interesting.

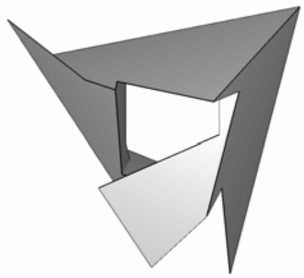
A sculpture (F 7 toroid)
in the Fermat's birthplace,
Beaumont de Lomagne, France

Conclusion: It is a lucky circumstance, when an abstract sculpture exhibit a significant mathematical achievement as well, such as in our case.

References

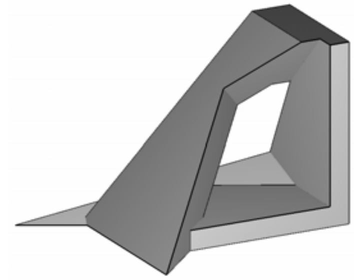
[1] L. Szilassi, Some Regular Toroids, *Proceedings of the 11th Bridges Conference*, Leeuwarden, 2008, pp. 459–460.
[2] L. Szilassi, *Locally Regular Toroids with Hexagonal Faces* Symmetry: Culture and Science Vol. 20. Nos.1-4, 269-295, 2009
[3] *Proceedings of the 11th Bridges Conference*, Leeuwarden, 2008, CD-ROM /Extras/Some_Regular_Toroids/

APPENDIX

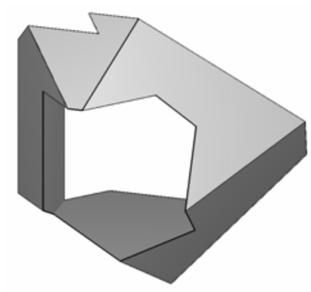


F13 A

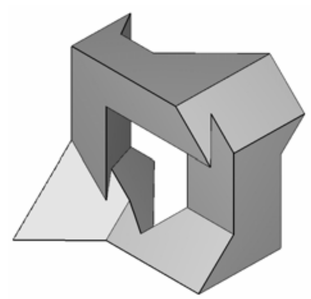
F13 B1



F13 B2



F14 A1



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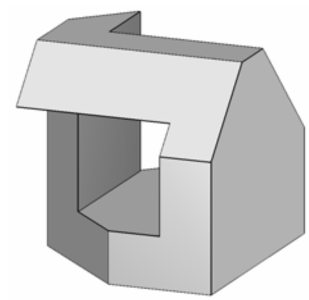
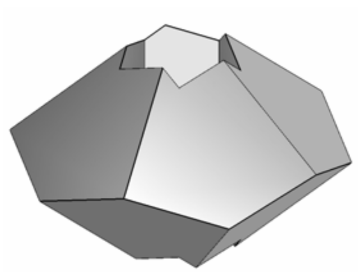
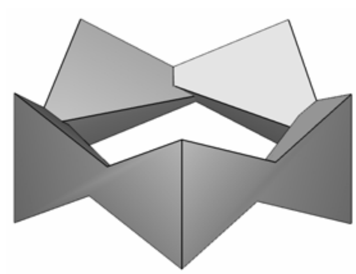


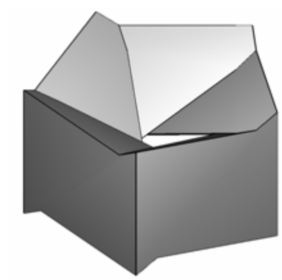
F14 B



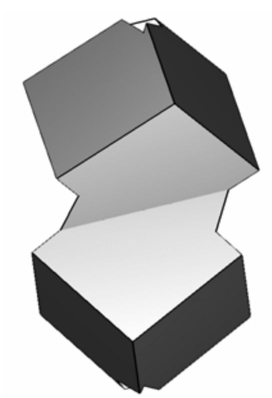
F15 A1



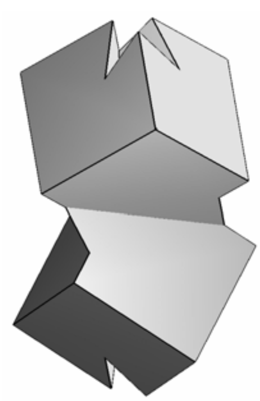
F15 A2



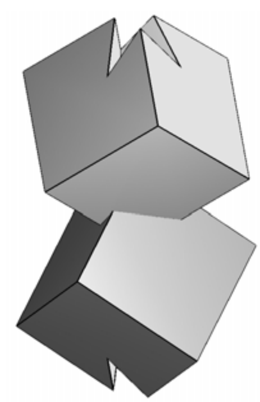
F15 B



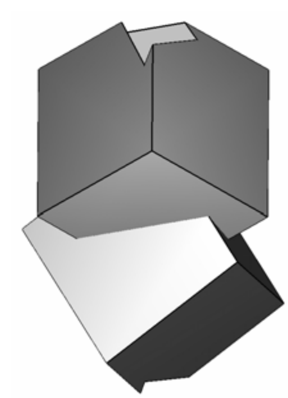
F15 C



F15 D1



F15 D2



F15D3