Escher Unraveled: Using Artwork to Investigate Transformations

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Abstract

Escher's artwork images are among the most identifiable of any artist. Upon first viewing Escher's artwork, many wonder "How could he have done that?" But once students understand the mathematics behind these images, they can decipher how they were created. In this paper, we describe an activity that accessibly intertwines mathematics and art for students ranging from middle school to undergraduate level.

Introduction and Context

This activity motivates students to deepen and apply understanding of plane transformations and properties of polygons. As such, it could be used in a variety of courses including math content courses for prospective elementary and middle school teachers. A novel setting would be for seminar classes that connect mathematics to art.

Steps of the Activity

In this three part activity, students constructed their own tessellations, then examined characteristics of their tessellations, and finally used this experience to analyze tessellations by M.C. Escher. Each student constructed a translated general parallelogram, a rotated equilateral triangle, and either a translated or rotated regular hexagon. All of the constructions were done using Geometer's Sketchpad software. See below (Figures 1–3) for three example pieces of artwork created by undergraduate students using these methods. For the first two constructions, students were given guideline steps to create a tile and tessellate it in the plane. For the third construction, students independently extrapolated ideas and applied them to transform a regular hexagon. A summary of the steps given to students is presented here for readers interested in incorporating this activity into their courses.

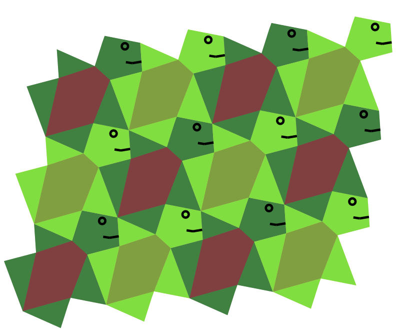

Figure 1. *Translated parallelogram tessellation.*

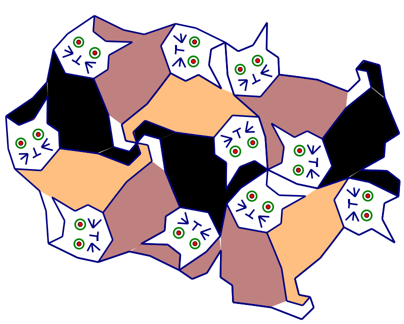

Figure 2. *Rotated triangle tessellation.*

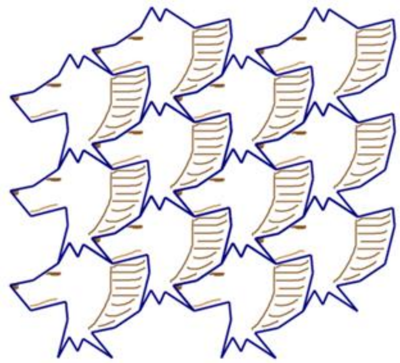

Figure 3. *Translated hexagon tessellation.*

For the first tessellation, construct a general parallelogram $ABCD$ and then create an irregular boundary along one side (which we refer to as AB). This boundary is translated by a vector along an adjacent side (AD) of the parallelogram to replace the side CD. Similarly, create an irregular boundary along side AD and translate it to replace side BC. The result is a tile modified from a parallelogram. At this point, use your creativity and artistic eye to move vertices on your original tile and then decorate by adding lines, circles, or other shapes in order for your tile to take on a recognizable form. The new shape is repeatedly translated along vector AB to form a row of tiles. Alternating the interior color of the tiles will make them easier to distinguish. The row of tiles is then repeatedly translated along the vector AC to fill the plane. Again, alternating the interior color of the tiles is useful to tell one tile apart from another.

In the second tessellation, construct an equilateral triangle JKL and create an irregular boundary along one side (which we refer to as JK) using connected segments that join J to K. Using point J as a center of rotation, this boundary is rotated by $60°$ to replace side JL. For the third side, construct the midpoint M of side KL. Then make an irregular boundary to replace KM by constructing connected segments that join K to M. Rotate this boundary by 180° about point M to complete the modification of side KL. The modified equilateral triangle can be adjusted as desired by dragging points of the new tile as long as none of the irregular edges intersect. Use your imagination to transform your tile into an interesting form. To tessellate the tile, rotate the new shape $60°$ about point J until there are six copies of the tile. Alternate the colors of the tiles to make each tile distinguishable. Now rotate the set of six tiles $180°$ about point M. Adjust the color of the tiles as necessary. Continue rotating sets of six tiles (point of rotation and angle of rotation is left for the student to determine) to tile the plane.

For the final tessellation students have the option of translating a regular hexagon or rotating a regular hexagon. Here the experience from the first two tessellation exercises is applied to a different base tile. Students must understand the principles behind their first two tessellations in order to extend ideas of transformations to the new context of the hexagon and decide how to modify a six-sided figure.

In the second part of the activity, students are directed to take a closer look at their tessellations created in part 1. For each of the three tessellations, they examine how many tiles appear around each vertex and determine how that number relates to the measure of the interior angles of the original tile. Then they compare the orientation of the figures in the three tessellations to examine the resulting effect of transforming by translation versus rotation. As instructors, we have found that when students are specifically asked to analyze their figures, they can more easily apply the ideas to new tessellations.

As a culminating activity, students become Escher sleuths by investigating the underlying shape (triangle, rectangle, or hexagon) and type of transformation (translation or rotation) in M.C. Escher drawings. Students can draw upon the analysis of their own tessellations to justify their classifications. A wide variety of Escher drawings illustrating these ideas is available on the web [1].

Acknowledgements

We would like to thank the following students for giving us permission to include their artwork: Shira Glushakow-Smith for the translated parallelogram tessellation, Zachary Claypool for the rotated triangle tessellation, and Jessica Tollman for the translated hexagon tessellation.

References

[1] A. Bart and B. Clair, Regular Division of the Plane Drawings (2012), http://euler.slu.edu/escher/index.php/Regular_Division_of_the_Plane_Drawings (as of Feb. 29, 2012).