Mathematics in the World of Dance

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Abstract

Over the years our society has considered dance and mathematics to be near polar opposites. Dance is a fun activity - both to perform and observe; it involves other people and can be source of great satisfaction. On the other hand, the general populace considers mathematics to be a dull and overly complicated source of constant frustration. The two seem to have nothing in common. And yet upon close investigation the many connections and similarities reveal themselves.

Introduction

Mathematics is present in dance. It is not the mathematics of simple number manipulation; we do not attempt to add or integrate through movement, instead we would like to employ abstract mathematics and various methods of analysis to understand dance at a deeper level. In this paper we would like to highlight a few basic examples of interplay between mathematics and dance.

Why we should care

Dance can be used to teach the fundamentals of mathematics and provide the students with basic intuition about the abstract concepts involved. Actually getting to experience math at work might be more exciting to students than "Two trains leave cities A and B going at 60mph…" Applying mathematics to more familiar 'real life' situations would certainly remove the stigma of the field being dry and inaccessible.

The math concepts can be used consciously to create dance. Many choreographers create pieces based on their intuition; being explicitly aware of the principles they are applying could help speed up the creative process. Taking the idea further, some might decide to structure the whole piece in terms of count and formation progressions, then within those constraints decide upon actions of individual dancers.

Finally, the awareness of how mathematics and dance interact and draw from each other can help us understand both areas on a whole new level and keep the inquiry exciting.

Geometry in Dance

Geometry is perhaps the most apparent subfield of mathematics present in dance. We can consider the shapes, patterns, angles and symmetry of many different aspects of dance within a variety of scopes. The analysis could concern anything from one dancer frozen in a position to a whole ensemble actively moving in space. In the first case, we would look at the lines of the body and their relation to each other and to the space in which the dancer exists. In the latter, we would consider not only the lines and shapes created by the collective and the way in which they change with the music, but also the patterns of beats bringing on those changes.

Figure 1: *A dancer holding a position.*

Figure 2: *A dancer in the middle of movement. Photo credit: Elston Photography*

Consider the dancer in Figure 1. She is in a grand plié in the second position turned out on relevé, hands straight above the head. Looking at her from the front, her legs and the floor form a rectangle, meaning that the thighs are parallel to the floor and the shins are perpendicular to the floor. Thus the angle at the knees is ninety degrees. Her arms form a V-shape, so that drawing an imaginary line connecting her palms forms triangle. The dancer's body is symmetrical about her spine – the left and the right sides of her body are mirror images of one another. Looking at the grand plié from above, due to the turnout of the thighs the dancer's body should follow as straight of a line as possible. Though, depending on the technical level of the dancer and physical constraints, the legs might form an obtuse angle.

Now, consider a dancer coming out of an attitude with layout – Figure 2. Her back leg is slightly bent; her upper body is arched back, with the spine perpendicular to the ground. The extension of the front leg forms the axis with respect to which the back leg and the upper body mirror each other. As a consequence the line passing through the front leg bisects the angle formed by the back thigh and the spine. In addition, the line tangent to the curve of the upper body at the hips dissects the angle formed by the dancer's thighs. The arms form an ellipse with the dancer's head being the lower focus.

As the number of dancers increases, so does the number of possible relations within the ensemble. We not only have the lines formed by the dancers individually, but also the shapes and patterns arising from their bodies interacting together. In the tango pose in Figure 3 the man's and the woman's bodies are in fairly similar arrangements; in fact one could construct the woman's pose from the man's by a sequence of simple transformations: reflection, rotation and rescaling. The rescaling causes the woman's pose to become shorter and wider – her back leg reaches farther than the man's back leg. In consequence she is exaggerating the movement to get lower, into the position of surrender typical for argentine tango.

Pieces involving more than one dancer very often use the idea of translation. To be more specific, if we asked a whole ensemble, or even just a few dancers within the group, to perform the same movement at the same time, we introduce translation of that pose. The geometry of the formation the dancers assume

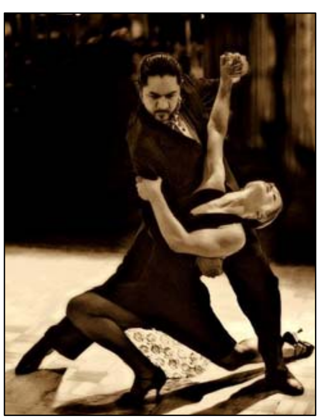

Figure 3: *Argentine Tango pose. Photo credit: Laura V Mingo*

Figure 4: *A ballet ensemble employing translation. Photo credit: Elston Photography*

Figure 5: *Quartet formation. Photo credit: Elston Photography*

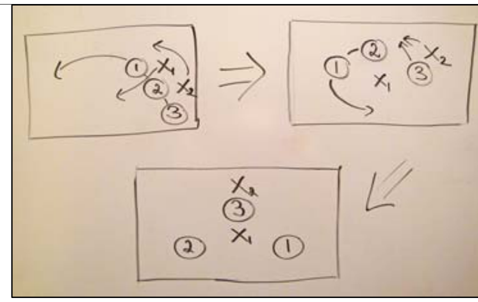

Figure 6: *Formation change.*

is independent of the shape of the pose; the choreographer could position the dancers in any arrangement he pleases. In the piece in Figure 4, the dancers form two parallel lines; the simplicity of the formation emphasizes the beauty of proper ballet technique. However, depending on the feel of the piece a choreographer might choose to place his dancers in a pyramid or a differently organized formation.

Since dancers are three dimensional creatures, their movements and poses exhibit different geometrical relations depending on the angle at which we are observing the piece. In addition, with groups, we might need to deconstruct the formation in order to find relationships. It is possible, and in fact more interesting, to have a pose that as a whole does not posses simple geometric properties, but when taken apart exposes their presence. In the photo in Figure 5, the two dancers on the outside are in the same translated pose; their bodies define a splitting line for the remaining two ladies. Those fill in the vertical levels while bringing the ensemble together through the shapes their bodies create. The dancer in the front has the same leg arrangement as the outside ladies; she is exaggerating the knee bend to get into her position. The dancer in the back is opposing the other's downward action. Seeing this formation from the side, we could see that the dancers' spacing forms a zig-zag like line on the dance floor while their bodies form another one in the vertical plane.

Changes in formations are the icing on the cake in the scope of dance geometry. The transitions embellish and complement any relations that were imposed on the formations between which they occur. Consider the image in Figure 6. We have five dancers with the ladies, represented by circles, aligned in a diagonal line with each of them dancing the same translated choreography. The men, the crosses, are initially staged right together. We would like the next formation to be a triangle whose lower vertices are two of the female dancers performing symmetrical movements. The top vertex would have a couple in it and the last man will be placed in the middle of the shape, so that the shape is nice and balanced. One way to go from the initial to the terminal formation would be for the middle man to set the triangle and the rest to spiral around him onto their respective positions. This particular change pattern also introduces an element of complexity and excitement, since the dancers first seemingly scatter around the dance floor before continuing into their predetermined static positions.

Geometry in dance is unavoidable. Above we have presented a few varying examples of the many levels on which one could look for geometric properties in dance. The moment a dancer enters the floor, their body and their moves create shapes and patterns that simply wait to be noticed by the audience.

Beyond Geometry

Geometry is not the only mathematics concept that has sneaked its way into the world of dance. Because of the simple fact that dancers change their positions in space as time passes, the ensemble can be looked at as a multidimensional dynamical system. We could consider each dancer's position in space as the elements and explore the system's behavior as time goes on. One could choose to explore how the position of each dancer evolves with time which would entail following the paths they travelled over the course of the piece. Alternatively, we could consider all the dancers together and look at the path travelled by the *center of attention mass* (CAM) of the ensemble. To calculate the CAM of the ensemble, instead of recording the body masses of our dancers, we would assign the weights based on the type of movement performed and how likely the moves are to attract the audience's attention. We might want to assign zero weight to the dancers that are off stage. Then, for example, a dancer leaping across the stage would carry more weight than a dancer frozen in a pose somewhere stage left. Or, depending on the atmosphere of the dance, a dancer crouching down and being still center stage left could have more weight than dancers moving around him. Consequently the weight of each dancer would vary throughout the piece, and so would the position of the CAM. We could define a function that keeps track of that position based on time and ask how fast and in what ways this function changes. The majority of choreographers, in fact, perform this exact task intuitively; they look at their formations and make sure that the arrangements don't feel *heavy* on any particular side; they also make sure the transitions feel fluid that the ensemble as a whole follows the predetermined progression path. The author plans to develop the concept of CAM further in subsequent work.

In addition to the above, we could take our system of dancers and look at how often certain types of movements appear. We would then be exploring the statistical properties of the piece. We could check what proportion of time the dancers were using suspended movements, movements on the floor, or jumps; we could consider how many sharp versus fluid movements there were, or how many fast versus slow movements were used. There is a delicate balance between the proportion of different types of movements and how interesting the piece appears. A piece in which the majority of movements are low to the ground without being properly balanced out by the high level movements would most likely seem boring. Clearly, the position of the most pleasing ratios on the spectrum of possibilities would vary depending on the subject of the dance and the effect the choreographer is going for.

Another interesting concept is patterns of rhythms and the changes within those patterns. Not all dances follow the simple one through eight. We could have a dance in which a couple of distinct count patterns get repeated, or even they themselves come in a pattern. For example, suppose that the observed dance uses only three types of counts – we denote the count varieties as *A*, *B* and *C*. *A* could stand for 1&2&3&4567&8, for instance. Then, we can analyze the frequency of individual count motifs and the way in which they interact.

Final Thoughts and Acknowledgements

We are at a point in history when the world is becoming increasingly integrated. Our society has become more open to the exploration of possible crossovers between seemingly unrelated fields. The link between mathematics and dance has existed long before people decided to inquire about it. The purpose of this paper was to demonstrate a few basic interplays between the two disciplines and thereby begin an investigation of the limits of the association and the advantages that might arise from this knowledge.

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