Mathematical Eyes on Figure Skating

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Abstract

The presenters will offer a workshop integrating mathematics and figure skating in the classroom. Figure skating is an Olympic sport as well as an art form that takes place on semi-frictionless ice. Activities in this workshop will focus on algebraic reasoning and geometric angles. Participants will learn steps and turns that can take place on the ice, and map videos of 3-dimensional skating programs onto 2-dimensional paper. Participants will also consider several different angles: the angle formed by a skater's blade and the ice surface, and the angle formed by the skater's two legs; and discuss ideal angles as they pertain to art. Lastly, participants will explore applications of conservation of momentum, a physical concept, as they relate to partnered skating. The goal of the workshop is to offer many opportunities for participants to see a mathematical point of view of a popular winter sport.

Mathematics and Figure Skating

Figure skating and applied mathematics both share the characteristic that there exist theoretical standards of the ideal while at the same time, practical constraints must be considered. The US Figure Skating association administers figure skating tests, and skaters who wish to pass these tests need to create the close-to-ideal patterns on the ice by adjusting their body positions and ankle pressure accordingly. This workshop extracts the relevant mathematics that is inherent in these figure skating tests and provides participants with middle school level mathematical activities that motivate the learning of mathematics in sports.

In figure skating, tracings are formed on the ice. Each of the skating steps can be performed on a figure eight pattern, hence the coining of the name of the sport. [1] In addition to the patterns that skaters' tracings make, figure skating is also judged on the aesthetically pleasing poses and skills that skaters demonstrate. Figure skating is done solo as well as in pairs teams; in this workshop we also examine the conservation of momentum when two people skate together.

The activities of the session will be done in groups such that each group of participants will have time to participate in each of the three following described activities.

Activity 1: "How many steps and turns?". In this activity, participants will be introduced to the kinds of steps and turns that can be done on a figure eight. Those participants who would like to try performing some of the steps and turns on the floor (on a big figure eight pattern) will be able to do so. Participants will generate a list of possible steps and turns that can be done using the right or left foot, forward or backward edge, and inside or outside edge on each of the two circles. This activity involves elementary combinatorial skills.

In the second part of this activity, participants will be shown videos of footwork performed by solo skaters on a rink surface. These skaters were filmed as a part of a US Figure Skating project to help coaches prepare their students for skating tests [2]. In the videos, skaters will be traveling in their three-dimensional space. Participants will then identify which kinds of steps and turns that these skaters are performing. Based upon participants' analysis of the video, and they will also identify what two-dimensional diagrams correspond to the three-dimensional videos. This visual challenge in identifying the one-to-one correspondence between skaters' actions and diagrams is a real-life use of functions! Skating judges must evaluate skaters' actions and how well they correspond to the ideal diagrams.

Activity 2: "Watch Your Angle". Elementary and middle school students often have accurate conceptions of angles when the angles are stationary (e.g., they can recognize that the wall forms an angle); however when angles are static, students may have trouble recognizing angles (e.g., a pair of scissors creates many angles) [3]. In order to assist students with recognizing that static motions that create angles, this activity highlights the angles inherent in physical motions.

In the first part of this activity, participants will evaluate photographs of skaters either skating or stretching, and will identify the angles formed by the skaters. Some participants will use protractors to determine these angles, and other participants will use trigonometric ratios based upon distance measurements in the photographs. Below are some examples of photographs which could be analyzed.

Figure 1: *Skater who is ready to enter a spin; angle of skater's leg to the ice highlighted*

Figure 2: *Skater who is ready to enter a spin; angle between skater's head, leg, and arm highlighted*

Figure 3: *Skater stretching.*

In the second part of this activity, participants will each "strike a pose" and will photograph each other in their poses. Each of the photographs will be imported into a Geometer's Sketchpad file. Using the tools (e.g., Measure, Calculate) on Geometer's Sketchpad, each participant will write up a report identifying all angles in their pose.

Activity 3: "The ice surface is frictionless". Because an ice surface is relatively frictionless compared to the ground, the ice is a suitable medium in which we can illustrate elementary concepts of conservation of momentum. We focus on two types of motion in this activity: linear and angular.

To model conservation of linear momentum, suppose that two pairs skaters individually each are skating separately and then they come together as a unit. If we know their weights and beginning velocity when skating separately, we can compute what their resulting velocity will be when they skate together. How much faster does the lighter skater need to push individually, in order to create a 25% increase in speed as a team? We will answer this question and other related question by examining the equation for conservation of linear momentum in an elastic situation.

To explore conservation of angular momentum, participants will experimentally test to see whether they spin faster on a plastic spinner on the floor, when their arms are perpendicular or parallel to their bodies. We hope that participants will gain an appreciation of the balance that skaters need to have on the ice, as well as question why they spin faster when their arms are perpendicular to their bodies. We will show a video demonstrating a skater spinning on the ice [4], similarly to the manner in which the participants were spinning on the floor, showing that the same result happens in the skater's spins. We will then demonstrate how the two scenarios (arms perpendicular, arms parallel) and the resulting angular velocity are related by the equation for conservation of angular momentum.

Mathematics and Other Sports

In this workshop, we examine figure skating as a sport that incorporates combinatorics, functional mapping, angles, and momentum as ways in which one can evaluate and improve skaters. Every sport has motions that can be modeled in a mathematical way. As teachers and parents often seek to motivate their students, using sports as an impetus for the learning of mathematics could help students learn to see the world through a mathematical lens. Similar activities as those presented in this workshop can thus be created for any sport of interest (e.g., [5], [6], [7]). We seek to raise mathematical literacy even within all of the avid sports enthusiasts!

References

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