Significance of Pointed Domes: Morphology, Typologies, and Geometrical Design

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Abstract

The aim of this study is to offer a closer view of pointed domes as the most outstanding and significant cultural features of domes in Iran and nearby areas. This study addresses the better comprehension of such domes, including their brief morphological features, three sub-typologies, and geometrical constitutions based on their shells' geometrical designs. Using the *al-Kashi* geometrical essences, a four-centered profile as an initial pattern is geometrically developed using new parameters to deduce the formal geometrical commonality of such domes. Despite various forms and configurations of Islamic domes, the research put forward some recurring features and shared traits that are incorporated into a specific style, that is, 'pointed domes' in Iran and surrounding areas. It can also be a basic geometrical approach for the derivation of the formal geometrical language of Eastern domes.

Introduction

Pointed domes, erecting on single and/or huge complexes, mainly reflect significant and important origin of bulbous domes in Iran and nearby areas. They have been well known due to their proportionally designs, graceful constitutions, various sub-types and unequivocal geometrical designs. Such a study simplified the enduring distinctive issues between the varieties styles of Eastern domes. Yet, it may lay down a solid ground for diverging geometrical values of these heritage edifices, resulting from the close collaborations between Islamic mathematicians and master builders.

Terminology

All too often individuals note the commonality of shapes of Islamic domes, despite considerable differences between their conceptual forms. Researchers have frequently used both terms of onion and bulbous to describe the type of domes under study that may create conflict in comparing their geometrical concepts.Unquestionably, the lower part of a pointed dome profile is geometrically tangent to two vertical lines passing from the end points of its span (Fig. 1a) as opposed to the profile traits of whether bulbous or onion domes (Fig. 1b). Owing to their profile considerations, the term of "pointed" as the most appropriate name comes from its dictionary meaning.

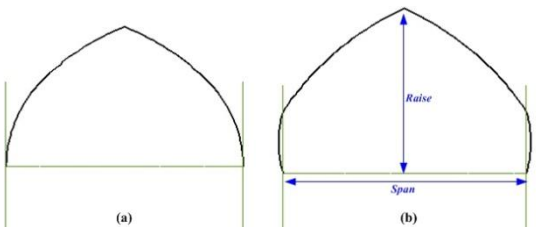


Figure 1. *Illustration of the main difference between profiles of pointed and bulbous domes. [Source: Author]*

Morphological Features of the Pointed Domes

Historically, the pointed style of Eastern domes rooted in long-term developments of both ideology and composition by various pre-Islamic civilizations and culture which inhabited this particular region. Chronologically, the earliest samples were associated with mud brick-corbelled vaults and domes in Mesopotamia and Egypt. But, theories about their origin and introduction into Islamic architecture can be ascertained by their isolated constructions in the 10th century. A great number of samples until the twelfth century demonstrated simultaneously architectural forms and geometrical compositions of such domes, if compared to other types of Eastern domes (e.g., conical domes) that were frequently constructed in this blooming period, *see* [1; 2]. Morphologically, the final configuration of the pointed domes resulted from the continuous development of four main components or "vocabularies": supporting system, transition tier, drum, and shell (s) (Fig. 2).

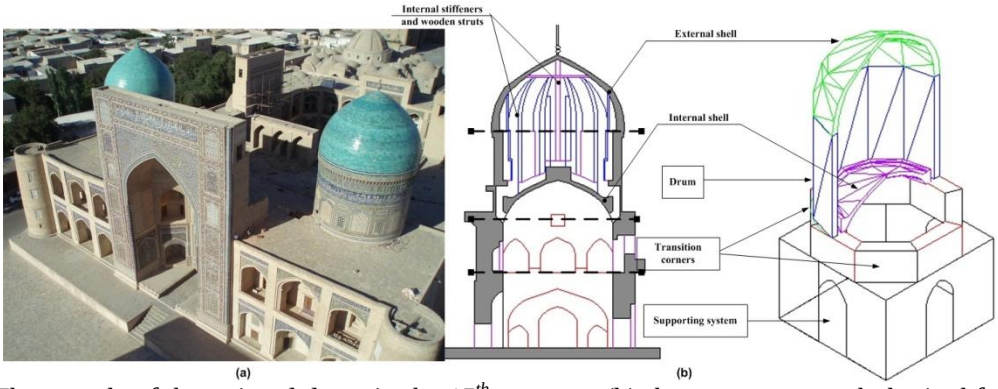


Figure 2. (*a) The sample of the pointed dome in the 15th century; (b) the common morphological features of the pointed domes. [Source: Author]*

The developed Geometrical Approach for the Derivation of Geometrical Prototype and Sub-types of the Pointed Domes

By analyzing a group of twelve samples, the striking resemblances are mutually manifested in the overall forms and dome's arrangements, especially, in the 15th century. Compositionally, the remarkable developments of their forms and compositions stemmed from the collaboration of Islamic renowned mathematicians in respect of geometrical and structural designs, such as *al-Kashi* who proposed five approaches for designing majority of domes' shells, arches and vaults, *see* [5](Fig. 3).

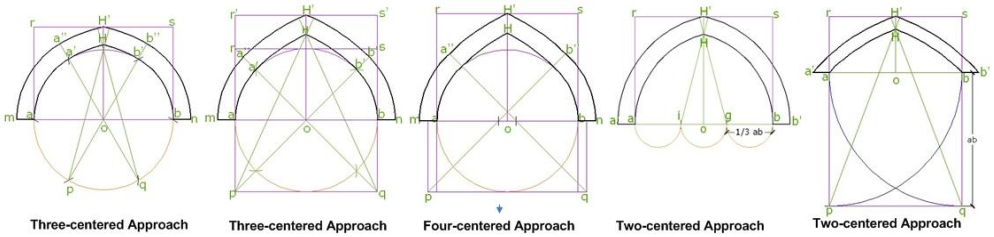


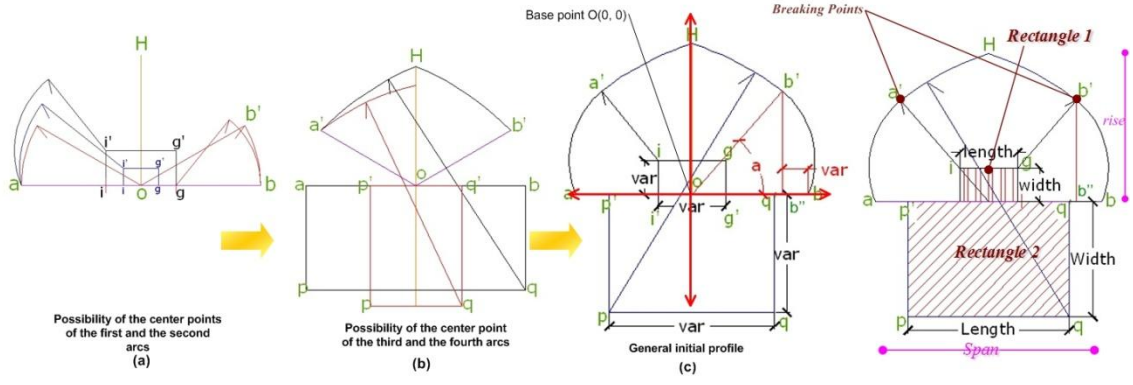
Figure 3. *The al. Kashi geometrical approaches from Kashani's Key of Arithmetic. After [Hogendijk and Sabra 2003; Dold-Samplonius 2000]*

The key in understanding any geometric designs of Eastern domes mainly bound up with studying, so-called '*profiles forms*' of their shells by diminishing shells' thicknesses. This profile consists of four

small arcs, named as the lower part (including first and second arcs) and the upper part (including third and fourth arcs). The geometrical parameters of this derived profile are explicated by two geometrical factors, 'span' and 'rise'. The horizontal distance between the two supporting members is called the span whilst the rise is the vertical distance from the middle point of the span line to the tip of the profile, *see* [2]. The span is entirely the origin and fundamental to all the rules for obtaining the value of proportions. In order to formulate a derived profile of any shells, it is generally necessary to determine three geometrical parameters; these are the center points of the first and second arcs, the center points of the third and forth arcs, and the positions of the breaking points. Firstly, a four-centered profile is developed as 'a general initial profile' of any dome contexts (can mutually be used for analyzing the pointed and bulbous domes) through the application of the former geometrical definitions (Fig. 4):

1. The center points of the lower arcs: are the loci of the points whose centers are located on the two vertices of rectangle *ii'gg'* constructed above the span line. Values of its lengths and widths are gained based on the exact proportions of the span. When *ii'*=*gg'*=0, then the centre points are located on the span line (Fig. 4a).
2. The positions of the breaking points: are the couple of points *a'* and *b'* used for changing the profile curvatures through two considered options; firstly, it can occur by crossing the perpendicular lines *a"a'* and *b"b'* from the points *a"* and *b"* which are marked from the end points of the span line based on the fraction of span ***ab*** (used for analyzing the bulbous domes). Secondly, the points *a'* and *b'* are gained from the certain values of the springing angles: <a=25', 30' and 45' (Fig. 4c).
3. The center points of the upper arcs: are the loci of the points whose centers are always set on the two vertices of the rectangle *pp'qq'* constructed under the span line. Values of its lengths and widths are also obtained based on the fractions of the span (Fig. 4b).

To facilitate the presentation and setting of these geometrical indications, a system is proposed as follows: {[*R*1], (B), [*R*2]} = {Rectangular1, (Breaking point), Rectangular2} whilst all values are calculated from the middle point of the span, so-called, O (0, 0), where *ab*=Span, and *i*=1, 2…5. Note that the string "var", which means 'variance' in the drawings, is a specific dimension or distance for having varied parameters on the specific direction. This function helps in the flexibility of the analysis and in defining the rules for generating common prototypes for the given sub-types profiles (*see* Fig. 5). The given lengths in this system are divided by two for obtaining the vertices *i*, *g*, *p*, and *q* of the proposed rectangles. In fact, the obtained vertices are symmetrically calculated and positioned on the both sides of the vertical axis (oH).



$$R_1 = \begin{bmatrix} ig = i'g' = \frac{m_1}{n_1}ab \\ ii' = gg' = \frac{m_2}{n_2}ab \end{bmatrix} \begin{matrix} (Length) \\ (Width) \end{matrix} \quad (B) = or \quad \begin{matrix} \angle a = 25', 30', 60' \, and \, 45' \\ (aa'' = bb'' = m_3 / n_3 ab, 0) \end{matrix} \quad R_2 = \begin{bmatrix} p'q' = pq = \frac{m_4}{n_4}ab \\ qq' = pp' = \frac{m_5}{n_5}ab \end{bmatrix} \begin{matrix} (Length) \\ (Width) \end{matrix}$$

Figure 4. *Illustration of the developed initial profile and its geometrical parameters.[Source: Author]*

Looking again at the primary definition of the pointed dome, to be tangent to the two vertical lines passing from the end of span, the centre points of the lower part (the first and second arcs) have to be set somewhere on the span line, meaning that the width of $R_1 = 0$ (Fig. 5). Furthermore, the sub-types of the pointed domes can also be categorized into three groups: shallow, medium, and sharp (*see* Fig. 5). This property fully conformed to the scale of the rectangle $pp'qq'$ (the rectangle under the span line) and the values of angle α . Note that the same method can also be adopted to derive the geometric prototype of the profiles of internal shells as well.

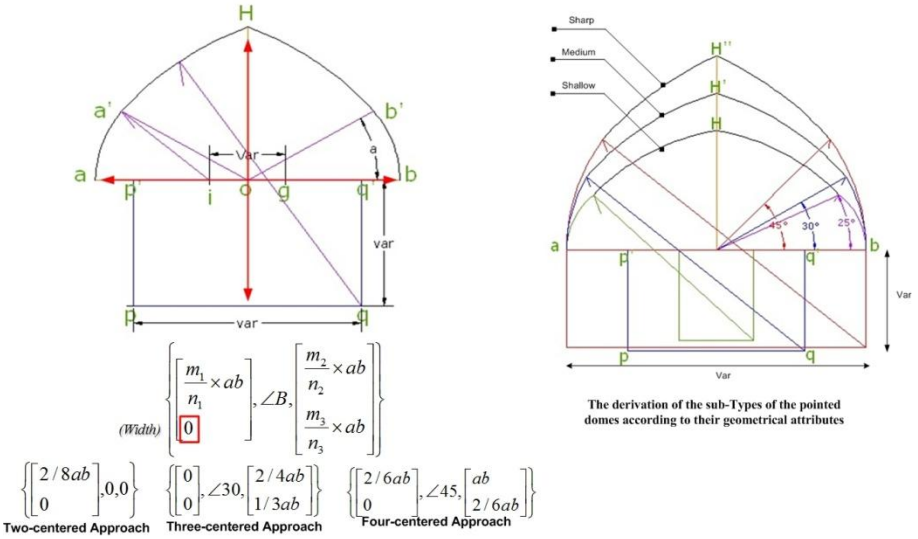


Figure 5. *Common geometrical prototype and sub-types of the pointed domes. [Source: Author]*

Conclusion

The present research proposed a new frame work, for the systematical identification of the architectural features of the pointed domes. Yet, it also offered the analytical understanding of their geometrical prototypes and their sub-types. Interestingly, this geometrical approach can be easily utilized by architects and non-architects to geometrically formulate any sorts of Eastern domes. The biggest challenge of this method was the creation of flexibilities in the configuration of the initial profile in relation to the locations of its breaking points. Theses points allowed the possibilities of covering both pointed and bulbous shells' designs. The obtained results systematically included two options for the breaking points.

References

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