Space-Filling Curves as Design Elements

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Abstract

The Mosaic Arts Center will be discussed and the teaching methods used to enrich children's learning of STEM (Science, Technology, Engineering and Math) concepts using Art will be demonstrated. To illustrate the teaching and learning program, a hands-on "Space-Filling Curves" demonstration using 3 levels of progressively more complex activities will be introduced. Color, Shape and Dimensions will be used in the demonstrations.

Introduction

Geometric concepts have long been used as design elements in art. Classical examples can be found in Escher's use of fractal structures in his "Smaller and Smaller" woodcut [1] and in Mondrian's use of rectangular grids in his painting "Broadway Boogie-Woogie" [2]. More recently, artists such as Janet Parke (especially in her piece, "Taupensky" [3]) and Kerry Mitchell (see, for example, "Hilbert's Ghost" [4]) have made fractals central to their work.

Art can be used as a method to present and teach technical content, such as mathematics. The art serves as an incentive to engage the student and as a concrete application of the concepts. These ideas will be explored in this workshop in the following activities. Students will learn about algebra, geometry, and fractals by using space-filling curves as elements in their own two- and three-dimensional designs.

The teaching concepts are incorporated in the Mosaic Arts Center and introduce participants to art play in the form of fractals in space-filled curves, explore their mathematical aspects, and experience them as design elements.

"Play is considered an important development tool in the total development of children. It is through play that children experiment and learn new skills, cope with mechanisms, solve psychological conflicts, obtain confidence and gain understanding of the things around them," states Andes [5]. With this concept in mind, the Mosaic Arts Center has developed a learning model that helps children learn STEAM (Science, Technology, Engineering, Art, and Math) through the development of Art play projects. Each project is composed of nine evaluated sections that are scored using a learning rubric. Topics include: Art, Cause and Effect, Measurement Skills (weight, length, liquid capacity, time), Project Management (vendor relations, time management, budget, scope of project), Research skills, Safety (work and personal), Spatial Dynamics, and Presentation Skills.

At the conclusion of the program the children will present their Art play projects in a competition to be judged for display in a public forum. Projects are developed based on the children's level of understanding (e.g., rudimentary shapes for younger students up through advanced analysis for older students) and comprise STEAM principles. These principles can be demonstrated math-based art projects, such as the activities in this space-filling curves workshop.

Activity 1: The Hilbert Curve

The Hilbert Curve is built up in iterations based on a 2 × 2 grid of squares. Each subsequent iteration includes four copies of the previous. When taken to its limit, the curve fills every point in the bounding square. The first three levels are shown in bold in Figure 1 (the lighter lines merely indicate the square grid).

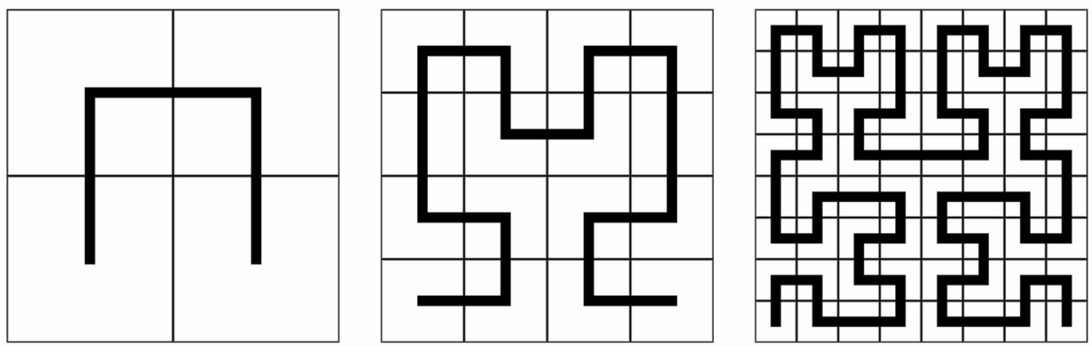

Figure 1: *The first three stages of the Hilbert curve.*

In this activity, participants will explore the aesthetic possibilities of the Hilbert curve. Examples in art will be shown, in which various techniques are employed, such as: combining multiple copies of the curve, using shapes other than line segments to render the curve, and coloring the foreground and/or background. (See Mitchell [6, 7] for examples of these methods.) After this introduction, the participants will be issued materials and set off to create their own works of art.

Activity 2: Two-Dimensional Extensions [

The Hilbert curve is an example of a self-avoiding walk, a polyline which never intersects itself. It is composed of horizontal and vertical segments one unit long, joining the centers of small squares within a larger grid. Relaxing one or more of these construction rules allows for the creation of many new and different types of space-filling curves. For example, the underlying grid could be 3 × 3 or of higher order. Alternatively, another base shape could be used, such as a triangle (see Figure 2; the triangular grid is only shown in the first case). So long as it can be decomposed into smaller, similar shapes, it can be used to generate space-filling curves. Any rep-tile (replicating tile) can be used; see Weisstein [8] for examples.

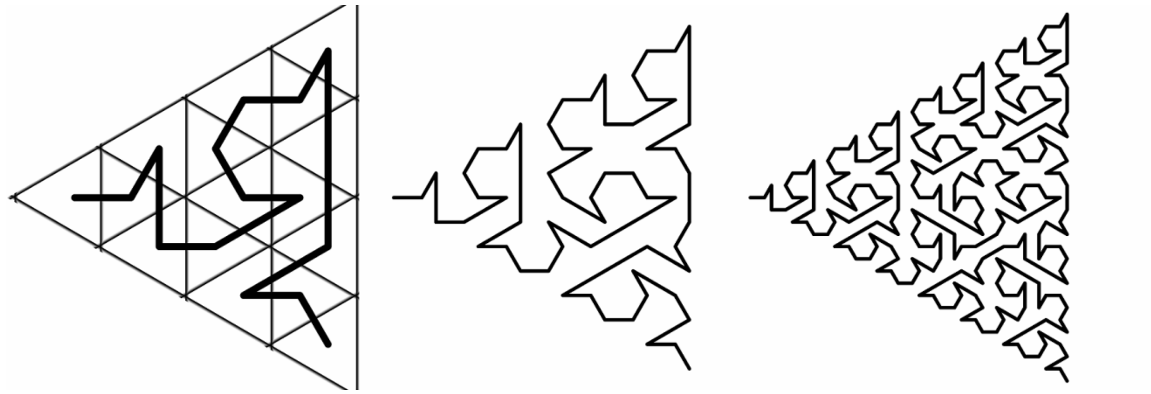

Figure 2: *Three levels of a triangular space-filling curve.*

Activity 3: Three-Dimensional Extensions

Volume-filling curves are three-dimensional extensions of space-filling curves, such as Gilbert's 3D Hilbert curve [9], whose infinite limit fills every point in a cube. Using similar construction principles as those for 2D curves (e.g., orthogonal segments, self-avoidance), such curves can be generated using a $2 \times 2 \times 2$ array of cubes, or larger. Figure 3 shows the first level of a curve based on a $3 \times 3 \times 3$ array.

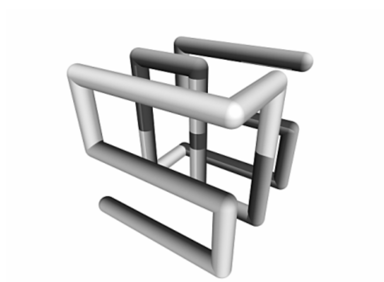

Figure 3: *First level of a volume-filling curve.*

Workshop participants will be encouraged to develop their own volume-filling curve, and then to construct a low-order model of it using supplied materials. They will then be challenged to find ways to employ these designs artistically. Some suggestions include simply coloring the model or using the model as a frame for a "string art" construction, as exemplified in Figure 4.

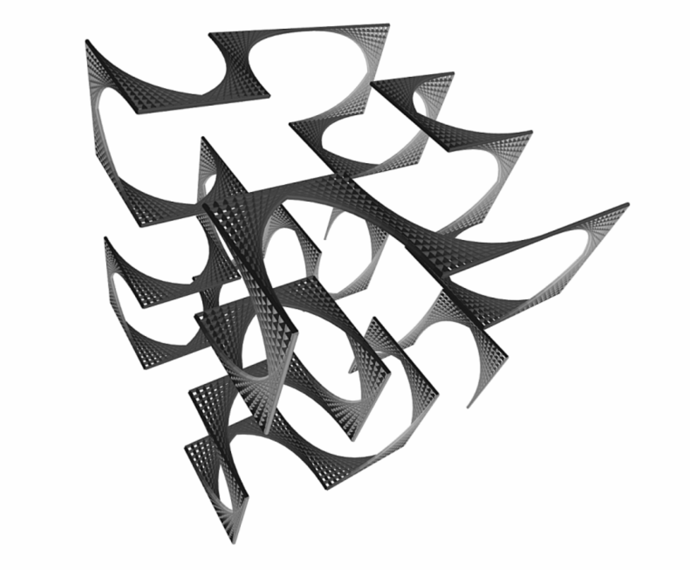

Figure 4: *Cube-filling curve rendered as string art.*

References

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