Models of Stellations of the Icosahedron

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Abstract

Stellation forms complex polyhedra from more basic polyhedra. The best way to study and appreciate these beautiful forms is to examine a three-dimensional model. I will discuss the geometry of the models that can be made from the pages of my new book, *Cut and Assemble Icosahedra: Twelve Models in White and Color* [1]. The models in this book are designed to allow one to construct complex polyhedra using the fewest steps possible. They allow both the mathematician and the lay reader to study the structure of these polyhedra in three dimensions.

Figure 1: *The Twelve Models from Cut and Assemble Icosahedra*

1. Introduction

The icosahedron is one of the five Platonic, or regular, solids. It consists of 20 equilateral triangles arranged so that five triangles surround each vertex. Given a polyhedron, such as the icosahedron, another polyhedron can be formed by extending the planes of the faces until the extensions intersect [2]. The resulting polyhedron is called a *stellation*. If the planes are extended farther and they intersect again another stellation can be formed. Of the five Platonic solids, the icosahedron forms the greatest number of

distinct stellations, an amazing 58 using the common definition [3]. The other four Platonic solids result in only four stellations between them. The complexity of these stellations is astonishing. Although there are many sources on the internet that allow one to see, and even spin, two-dimensional images of these stellations, the best way to truly understand the structure of these beautiful polyhedra is to build and examine a three-dimensional model.

2. The Models

Cut and Assemble Icosahedra: Twelve Models in White and Color is printed on cardstock that can be cutout, scored, and then glued to form the polyhedra. The models formed are the icosahedron and four of its simplest stellations. They are colored several different ways to emphasize different geometric properties.

The faces of the icosahedron are colored in ten colors with opposite faces the same color. This same arrangement of colors is used for the faces of each stellation so that the viewer can easily see how each model relates to the icosahedron and to the other models. The models are also designed to be the actual size that would result from extending the facial planes of the icosahedron. This makes the stellation process easy to visualize by comparing models.

Five models in white allow one to appreciate the overall structure without the distraction of the coloring. Two other models are colored to reveal that the second stellation of the icosahedron is also the compound of five octahedra, and the third stellation is the compound of ten tetrahedra. That these compounds arise from the stellation process is truly astounding.

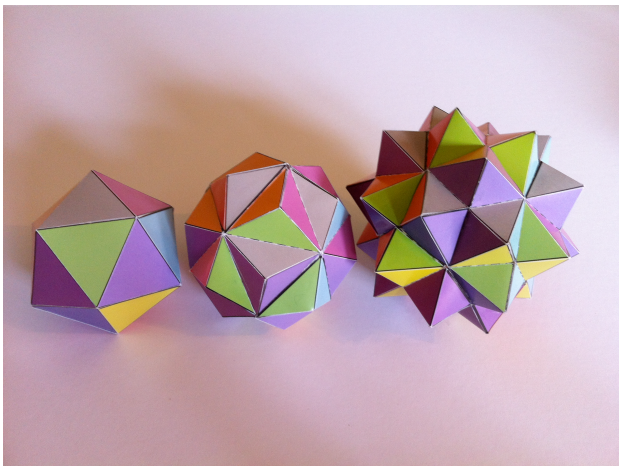

Figure 1: *Models are colored to emphasize the stellation process.*

References

[1] E.A. Torrence, *Cut and Assemble Icosahedra: Twelve Models in White and Color*, Dover Publications, New York, 2011.
[2] M.J. Wenninger, *Polyhedron Models.* Cambridge University Press, New York, 1989.
[3] H.S.M. Coxeter, et al., *The Fifty-Nine Icosahedra,* University of Toronto Press, 1938.