A Workshop on Making Modified Truncated Icosahedra Using 4D Frame

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Abstract

In this workshop we will show how to make modified truncated icosahedra using 4D Frame tubes. It starts from pentagonal pieces in a form of multiple frame and assembles it in mathematical patterns. Knowledge of the geometric structure is fundamental to understanding how these beautiful spheres are designed and constructed. Participants will experience mathematical creativity together with building and designing. By building these spheres, participants will gain a better understanding of three-dimensional structure.

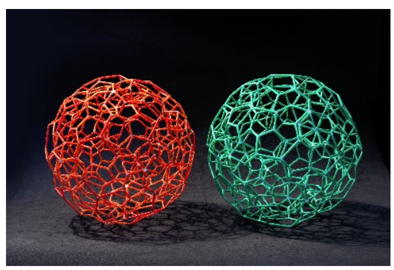

Figure 1 : *The Park-Sphere*

1. Mathematical Motivation

In geometry, the tesseract[1], called an 8-cell or regular octahedron, is the four-dimensional analog of the cube. A generalization of the cube to dimensions greater than three is called a hypercube. The tesseract is the four-dimensional hypercube, or 4-cube, and can be constructed in a number of ways, in particular, as a regular polytope with three cubes folded together around every edge.

Recently, the Dimensions Web site[2], which has awarded Prix d'Alembert in France, 2010 provides interesting short-films about dimensions. They cover many dimensions from 1 dimension to higher dimensions together with mathematical simulation and beautiful mathematical objects, such as wonderful animations of the 8-cell, 120-cell and 600-cell. In the films, there are stories about the fourth dimension - six regular polyhedra in dimension 4 which Ludwig Schlafli has described. Some are amazingly rich and the film proposes to show them to 3 dimensional spectators. Especially interesting are some patterns that are the shadows of four dimensional polyhedra in three dimensions and which excited our curiosity and imagination as in Figure 2.

Hence we want to extend two dimensional polyhedra to three dimensional artistic object. The Stellation Inspired Sculpture[3] given by E. Torrence in Bridge Coimbra 2011 led us to choose a frame constructing the sculpture with a connection to many fundamental frames.

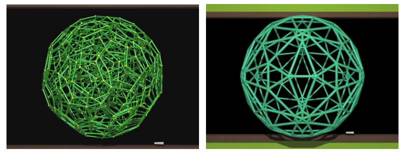

Figure 2 : *A shadow of 4-dimensional polyhedra 600 cell*

Especially the changing objects from 600 cell in dimension 3 as shown in Figure 2[2] have crossing connection in three dimension as distinct from two dimensional polyhedra. This phenomena invoked the way that connecting fundamental frames and modification mentioned in the E.Torrence's Sculpture Paper can be mixed to make a new, mathematical object in dimension 3.

A meeting with plane and space! We guessed that it is possible to connect fundamental plane frame into three dimensional fundamental solids in meaningful ways.

The truncated icosahedron can be constructed from an icosahedron with the 12 vertices truncated (cut off) such that one third of each edge is cut off at each of both ends. This creates 12 new pentagon faces, and leaves the original 20 triangle faces as regular hexagons. It is well-known that twelve regular pentagons can be coupled into truncated icosahedron similar to soccer balls as in figure 3.

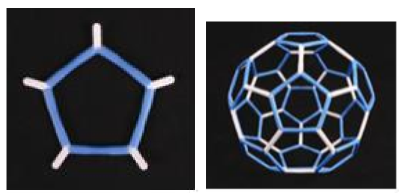

Figure 3 : *Truncated icosahedron and its fundamental frame*

We have wondered what happens if we change regular pentagons into other forms with crossing connection in three dimensional space. This question give us insight into the 3-dimensional mathematical model called *Park-Sphere* as in figure 4. The first author Park Ho-Gul has invented the material to construct as in the form of tubes. He called them 4D Frame.

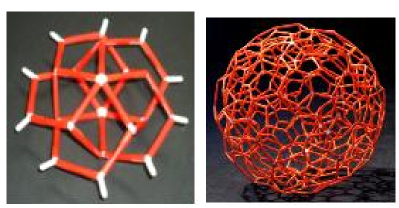

Figure 4 : *Park-Sphere and its fundamental frame*

2. Assembly Instructions

In this workshop, we prepare enough tubes for participants to join the assembly of modified Spheres. In general, it takes over 1.5 hours to complete Park-Sphere by oneself. The process of assembling Park-Sphere consists of 2 steps. Since we should make 12 fundamental frames in the step 1, we form groups of two or three persons so as to save the time to construct fundamental frames. The first step is concentrated on making the Park-Sphere fundamental frame as in Figures 5 and 6. One can similarly make fundamental frames in the Park-Sphere as pentagonal faces in the truncated icosahedron. The main difference is to connect each 4D Frame tube stereoscopically to obtain the right-side photo in the Fig 6.

Figure 5 : *Making fundamental frame in the center part*

In connecting sub-frame tubes in Figures 5 and 6, one should follow the position to upward part together with one specified direction if possible. The 5 sub-frame tubes in a pentagonal shape are assembled to keep crossing each components (sub-frame tubes) around the center tube.

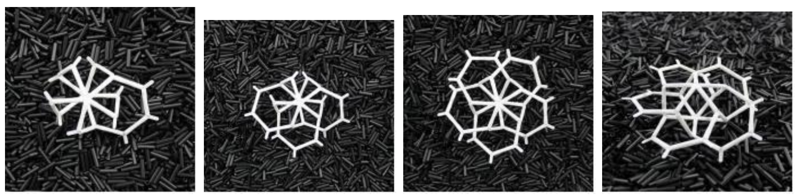

Figure 6 : *Making fundamental frame in the surrounding part*

The second step is dealt with by connecting each small piece to Park-Sphere thoroughly as shown below in Figures 7 and 8. After constructing 12 fundamental frames, the participant follows the procedure such as the connecting method through which the truncated icosahedron is constructed.

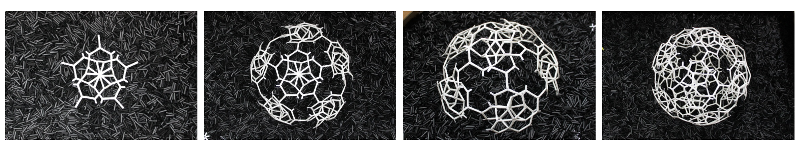

Figure 7 : *Connecting each fundamental frames to make sphere*

First one assembles the upper-hemisphere part with 6 fundamental frames. Fix one fundamental frame in the center. Around this, connect 5 fundamental frames in one vertex and then construct the lower-hemisphere part with 6 fundamental frames similarly. After attaching the upper-hemisphere part to lower-hemisphere part, one obtains the right-side photo in Figure 7. Then one can find 12 holes in the assembled sphere.

To confirm the position of holes and explain the connecting method, we choose a red tube to fill the hole as the first and second photos in the Figure 8 shows. Here we have to notice that these holes have two types, each with three directions as in the degree 120. One tube has long-length and the other has short-length.

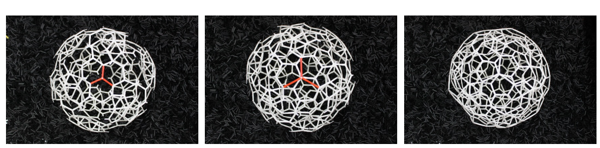

Figure 8 : *Attaching each tubes to fill holes*

3. Conclusions

Self-assembly activities get in touch with mathematical insights in a fun, hands-on way. Although the assembly is very prescriptive, the instructor can use discovery learning to guide participants in understanding similar mathematical structure through geometric approach. Eventually some participants can apply these principles in their own designs. Also, this kind of activity using 4D frame tubes can be applied to educational classes in developing creativity[6]. For example we can make the following new spheres as in Figure 9.

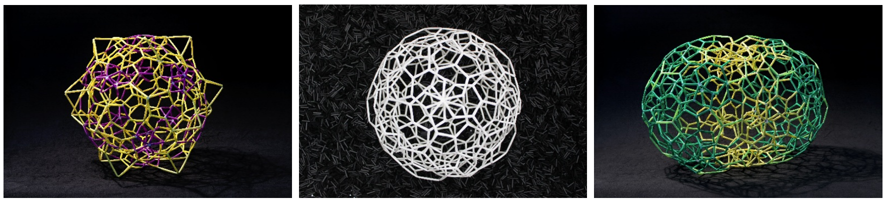

Figure 9 : *Modifying Park-Sphere to new spheres*

References

[1] http://en.wikipedia.org/wiki/Tesseract

[2] http://www.dimensions-math.org/Dim_fr.htm

[3] Eve Torrence, *A Workshop on Stellation Inspired Sculpture*, Bridges Coimbra, Bridges Conference Proceedings 2011.

[4] Park Ho-Gul, *The 3rd Soil, 4D Frame*, 1st ed. (2006), 4D Land Inc.

[5] http://www.4dframe.com

[6] Anneli Hedkvist Manninen and David Ostlund, "4D Frame-a new pedagogical material! A practical study", Spring 2010, Sodertorn University.

The invented Park-Sphere was certified on the register of the Korean Intellectual Property Office as in the registration number of 30-0558892.