Teaching Temari: Geometrically Embroidered Spheres in the Classroom

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Abstract

This paper discusses the various potential pedagogical benefits of using temari balls, or geometrically embroidered spheres in a mathematics class, either to craft or as objects of investigation. The corresponding workshop is designed to teach participants the basic techniques for crafting temari. During the construction process, the group will discuss ways to utilize temari balls to enhance student understanding of a variety of mathematical concepts.

Introduction

The use of art to pique student interest while providing an avenue to discuss mathematical content has become a common technique amongst a segment of the teaching professoriate. This is evidenced by the Mathematical Association of America Geometry and Art (2008, 2011) Professional Enhancement Programs (MAA PREP Workshops) [3,4] and by the long running, wildly successful Viewpoints: Mathematics and Art (2000-2007) by Annalisa Crannell and Mark Frantz, [2]. At the same time, a current pedagogical trend is toward student-centered classrooms. This managerial style allows learning that is simultaneously intellectually and physically active, such as the making of (part of) a temari ball or the small group study of a set of such balls. That is, the pedagogical atmosphere in classrooms has come together with instructor attitudes to yield favorable conditions for the use of temari balls in class.

We currently envision two distinct uses of temari balls in mathematics courses. In the first, students construct temari balls. In the second, students investigate existing sets of temari balls. We explore the potential of these activities below.

Constructing Temari Balls

The Japanese art form of the temari ball has modern realization as a thread-wrapped ball embroidered with a pattern. See Figure 1, left. Of primary interest to mathematicians are those spheres endowed with patterns exhibiting spherical symmetry. In order to accurately stitch such patterns, craftspeople mark congruent regions on the surface of the sphere before embarking on embroidery, as shown in Figure 1, right. This situation is reminiscent of workers preparing to decorate spherical domes, as described in [7].

Typically the preparation of the base sphere involves plotting the vertices of a projected convex polyhedron with regular faces and vertices lying on a sphere. We call the central projection of this polyhedron onto the sphere a spherical polyhedron. During this process, which plots points with pins and uses paper tape and pins to serve as straight edge and compass, many basic geometric definitions and facts arise for discussion. For example, locating antipodal points is a goal; therefore, knowing when one has found a great circle, rather than merely some circle on the sphere is of paramount importance. Similarly, knowing that an equator relative to a point (a pole) is found by plotting points a quarter of a circumference from the pole, rather than just equidistant from the pole, can be an important teachable moment for students, who do not grasp this intuitively or mathematically. Other geometric basics, such as copying and bisecting angles are natural points of discussion. Plotting the vertices is discussed in [7], [8], and [9], while detailed step-by-step descriptions of the process, including pictures, are contained in [9]. The subsequent step of wrapping great circle guide threads, known as guidelines, to mark the edges of the corresponding projected polyhedral is also contained in [8] together with three potential embroidery patterns. Patterns can be deliberately selected to highlight particular features, such as shown in Figure 2. In the left image, squares were formed by embroidering on every fourth thread of sixteen, whereas in the right image a five pointed star was formed by embroidering on every fourth thread of ten until the beginning was reached again. This application of orbits in Z modulo (n) indicates the types of so-called stars that can be formed on a temari ball with a certain subdivision at a vertex. When using in a lower level math course, one may want to pair this application with the Vi Hart video [5].

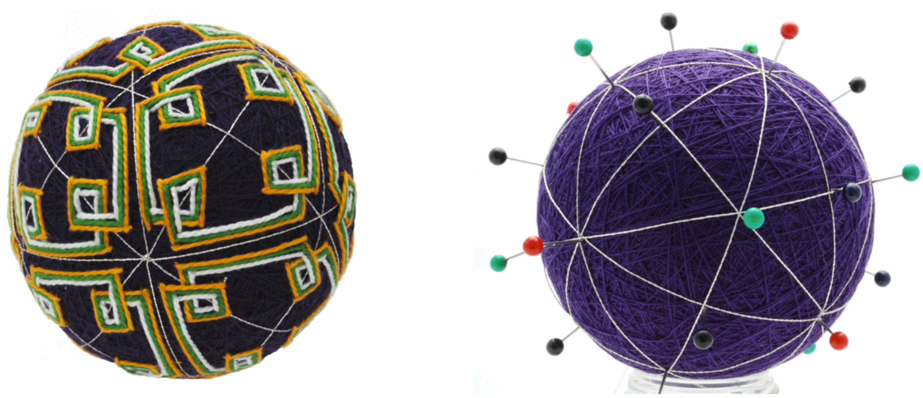

Figure 1: *A temari ball embroidered with finite spherical symmetry r432, left, and a temari base wrapped with thread and guidelines, ready to embroider, right.*

Figure 2: *Two temari balls showing rotationally symmetric motifs. Embroidered on a background divided with D_{16} symmetry is a motif with C_4 symmetry, left. On right, the center star is embroidered on a background with D_5 symmetry and the star has D_5 symmetry, disregarding thread crossings, or C_5 symmetry otherwise.*

As stated in [9, Theorem 1], the in-circle guidelines of a spherical polyhedron with regular-polygon faces produce a set of guidelines for the dual spherical polyhedron. For some projected polyhedron with regular faces, the usual guideline markings also form a set of guidelines for the dual polyhedron. For the others, additional great circles bisecting some of the angles can be added to the usual markings to form a set of guidelines for the dual polyhedron. The points of multiple intersection of the guidelines of high degree correspond to vertices of either the polyhedron or its dual—both are represented on the temari ball! See Figure 1 right, in which the green pins mark the cube vertices and the red pins mark the octahedral vertices. For students in a liberal arts mathematics class, this deep result can be simply observed as, "Each of these temari balls exhibits both a Platonic solid and its dual." Students can be asked to find evidence of this by counting faces and vertices. This becomes more visceral for students embroidering on a temari ball, who realize that the twelve stars they are currently stitching indicate twenty triangles to be stitched in their future. This combinatorial play can be augmented by having students calculate the number of stitches involved. At the same time, a straightforward examination of the temari guidelines allows students to observe that the sum of the angles of spherical triangles is not 180 degrees. This fact is particularly apparent upon inspection of the guidelines of the projected octahedron. Thus, geometry, elementary combinatorics, and Platonic solids (or other polyhedra), and number theory are all topics enhanced by the creation of temari balls.

Investigating Temari Balls

Many readers will have observed that students did not need to create temari balls in order to investigate their spherical geometry, combinatorics, and polyhedra. Because each temari takes a significant amount of time to craft (four to five hours for a simple ball), crafting time must be carefully balanced with time spent on mathematics. Sufficiently observant students can benefit from close inspection of a set of temari balls without having to craft any balls at all. We envision students guided by inquiry worksheets. The students may determine the underlying structure of the projected polyhedron (and its dual, if one is apparent), verify vertex, edge, and face relations, or find the finite spherical symmetry type displayed by the embroidery pattern. See [1] for a discussion of the fourteen finite spherical symmetry types, all of which are depicted in temari in [9]. Upper level students can be challenged to find innovative spherical tilings with various properties. Follow-up questions can revolve around plotting spherical points relevant to creating those tilings in temari. Examples involve plotting the projection of all Archimedean and Catalan solids. This author has a process for plotting the projected truncation of a solid when given directions for plotting the projected solid. Truncations are partially explained in [9]. Together with the projection of the Platonic solids, explained in [7], [8] and [9], this gives seven of the Archimedean solids and the corresponding seven Catalan solids, not all of which have been realized in temari by this author to date.

A related problem involves having students study pictorial representatations of guidelines. Students can be asked to determine how to plot those guidelines, what underlying polyhedra correspond to the depicted projected spherical subdivisions, and what motif decorations would appropriately take advantage of the guidelines. The instructor should be aware of the highly nontrivial nature of these questions before assigning them.

Value of Temari Ball Use

While temari balls are beautiful, and an investigation makes for a lovely diversion during class, will the students' intellectual gains offset the cost of precious class time, or is this just a pet project that the author has cleverly justified incorporating into class? In considering these important questions, we turn to John Mason in [6], who labels students as novices if they are at the stage where they are only able to notice or recognize connections (here between mathematical content and art) if they are clearly pointed out by someone else. Therefore, novices are unable to formulate such observations themselves. This is vital information for instructors, who might otherwise expect students newly having studied a topic to observe that concept in art without prompting. Our expectations are formed by our own training in finding examples of mathematics in the world around us. This training makes us unusually adept at locating tangible examples of abstract concepts. Those uninitiated into this quest for examples are unfamiliar with the strain of fitting (checking) increasing layers of abstract conditions onto a prospective tangible example to determine its worthiness.

We recognize that not all students will be sophisticated enough to engage in "noticing" examples for themselves. We consider three levels of students separately. Students of a lower sophistication level may not be able to check conditions verifying that an object qualifies as an example. These students can benefit from the invaluable geometric discussions that emerge when plotting points for creating temari balls. Temari ball use for these students needs to involve a great deal of time. Course goals must be broad enough to include basic geometry, which can come under the guise of rigorous mathematical thinking and justification. Note that some in the lower sophistication group may not achieve sufficiently positive gains from creating or investigating temari to justify use of the balls. Students at the middle level may be just starting to learn to find and verify examples. The use of temari balls can be worthwhile if only to exhibit this process. However, we believe that finding tangible examples of mathematical concepts helps lodge those concepts in long term memory and make those concepts available for transfer to contexts outside the initial realm of study. For students above the sophistication level of condition checking, providing visible, tangible examples for investigation can spur creative thought and deeper understanding. When deciding on temari ball use in class, one must determine whether the desired learning goals fit the potential learning goals for the sophistication level of the audience. In sum, the use of temari balls as a teaching tool to liven and reinforce student learning should be considered in a variety of mathematical situations.

References

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