Geometry and Art with a Circle Cutter

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Abstract

This paper draws attention to a method for constructing polyhedra such as the Platonic solids from circles. The construction method naturally segues into exploring and reinforcing various geometric concepts from the elementary to the university level. Practical, pedagogical and aesthetic reasons for using a circle cutter to construct polyhedra are discussed.

Introduction

In October 2011 I had the pleasure of being the mathematics expert for the two day conference *Forging Connections* held at the Calgary Science School. The goal of the conference was to establish connections between Science and Art for students in grades 3 to 8. Information can be found at [1] regarding the conference. The artist I worked with suggested an activity based on George Hart's H-construction (which can be found at [2]). A discussion of polyhedra was part of the activity. The experience taught me that making polyhedral models can be exciting for students and it taught me size (the bigger the better) and color (multiple colors being most attractive) are major factors in student enthusiasm about polyhedral models.

Many construction methods for polyhedra are known but I believe the method noted at [3] and [4] should be brought to the attention of educators. This technique uses folded circles glued together to create polyhedra and the construction method can be used to motivate geometric concepts at various education levels. Using a circle cutter speeds up construction and doesn't force the mathematics. Students could make the models in an art class or as seasonal decorations and later learn of practical geometry questions related to their constructions. Benefits of the method include:

- One person can mass produce a large number of pieces relatively quickly and the construction method is somewhat forgiving of minor construction errors.
- You control the size and color of the polyhedra produced through your selection of paper and circle radius. The resulting models are reasonably sturdy when made from cardstock.
- You have the option of making your polyhedra with decorative flaps. This is particularly striking on icosahedrons and other elaborate models made from triangles. (See Figures 1 and 2)
- It motivates a discussion of geometric concepts that you would not naturally consider when using other construction methods such as a polyhedral net.

The Circle Cutter Construction Method

I will briefly describe how to make a Platonic solid by this method. I have modified the method covered in detail at [3] and [4] to use the circle cutter, but their explanations are worth examining. After one understands how to make the Platonic solids, one can then figure out how to make other polyhedra (whose faces are regular polygons) from looking at a net for the polyhedra or pictures of it. Teachers can decide what solids to make, what stage students should take over construction of the models, and what roles art and geometry will play in the activity. The method's steps are, generally:

1. Select the Platonic solid you wish to make and select a radius for your circles. The circles will be folded into the faces of your Platonic solid. Cut out the required number of circles in the desired colors of cardstock. Also cut out at least one extra circle.

2. Inscribe the appropriate face shape (square, equilateral triangle or regular pentagon) on one circle. Locate the center of your circle by folding it in half and then in half again. You have centered your template circle on an x and y axis. Connect the points where the axes meet the edge of the circle to make a square. Reference [5] notes a nice trick to inscribe the equilateral triangle into the circle by folding alone. Otherwise take a protractor and measure out multiples of 60 degrees from one axis to locate the vertices of a triangle on the circle's edge or multiples of 52 degrees for a regular pentagon. Cut out the shape of your template polygon. After the template is made, lay the template over each circle in turn and trace your shape onto each of the circles. This can be done quite quickly. Go over the shape with a ruler and a somewhat sharp instrument like a nail to make the folding easier. Fold all the flaps. Your pieces are now ready for any decorating and gluing ─ the flaps are the tabs you will be gluing together.

3. Using a polyhedral net and picture of the solid as a guide, take your faces with flaps and glue them together. You have the option of gluing the flaps inside the object, gluing them so they stick out as flaps or gluing them on the outside as decorations. Pictures are below.

Figure 1: *The Platonic solids and an icosahedron and octahedron with flaps.*

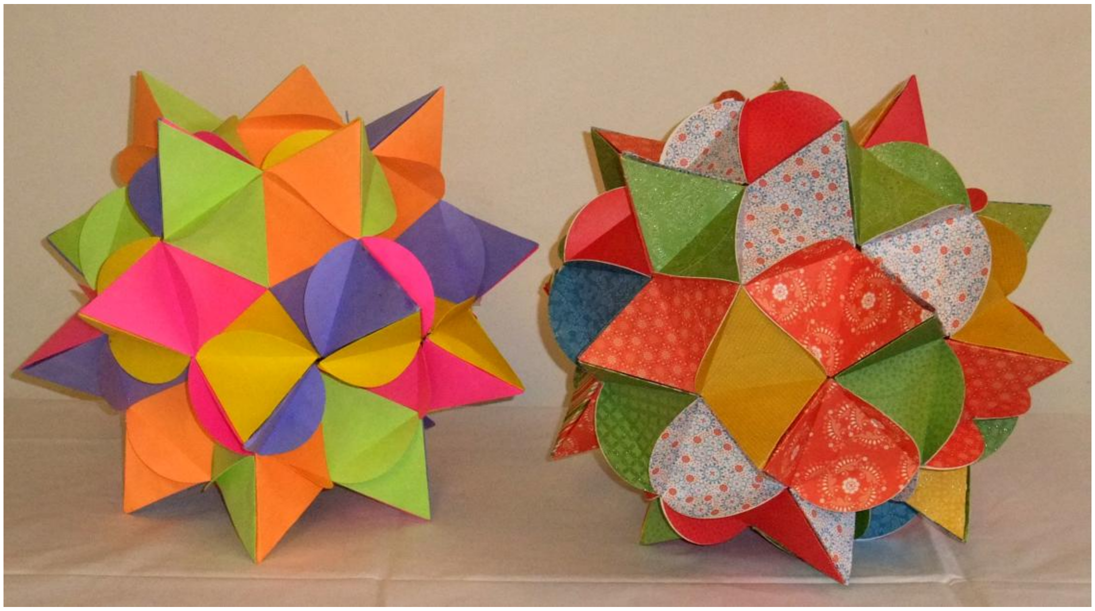

Figure 2: *The Rhombic Hexecontahedron discussed in [6] (with some flaps) and a variation of it. Each is constructed from 120 circles. It has potential as a group project, but each took over 3 hours for one person to construct from scratch.*

Pedagogical Advantages and Some Possible Geometric Investigations

Students find the models attractive and will see the construction of such shapes as a worthy application of geometry. Teachers can choose to put the math in right away or leave it until after the model construction. Also this method naturally utilizes the idea of inscribing regular polygons in a circle and this is a fruitful source of geometric discussions.

Below are a few ideas for geometric discussions directly related to the circle cutter construction method. I will not attempt to link the discussion ideas below to the curriculum in any particular grade/level.

1. One can start a geometric discussion in the earliest stages of the project. What basic property of a circle is a circle cutter based on? How can one locate the center of a paper circle? How can one inscribe a square in the circle? How about a regular pentagon? What is a regular polygon and how can we check our template shape before proceeding?

2. During construction one can appreciate the defining properties of a Platonic solid (face shape and angles). I believe these properties are easier to appreciate when the model is constructed piece by piece as opposed to by a net. If one has regular hexagons on hand (which you can make from circles) students can understand, through experiment, that one can't form a Platonic solid from hexagons or n-gons with n larger than 5.

After a model has been successfully constructed more geometric discussions can take place.

3. Consideration of the template piece and how it was constructed can lead to a discussion of inscribing regular polygons in circles, what the interior angles for any regular polygon are and how each polygon can be thought of as being composed of isosceles triangles. One could go on

to explain the area formula for a circle. One could also discuss ruler and compass techniques for inscribing regular polygons in circles. Explaining why the folding technique discussed in [5] generates an equilateral triangle would be a higher level investigation. Many geometric texts such as [7] discuss inscribing polygons in circles and could be used for inspiration for the further geometric investigations.

4. The Pythagorean Theorem can arise in a few different ways. Firstly, if one understands how to make the cube from circles you can ask what circle radius should be selected to get a side length of 6 inches for the square faces. Alternately one can ask: I wish to make a cuboctahedron (and show them a picture and net for the object). What do I need to do this? Students should realize they need both equilateral triangles and squares (and how many they need) and that the side lengths must be the same. One then can ask: "Suppose the equilateral triangles have side length 6 inches, what radius do I have to set the circle cutter at to make the squares?" Figuring out the side length of an equilateral triangle from the circle radius (or vice versa) would be an application of the Law of Cosines.

5. Area and surface area are other natural lines of investigation. How much paper was wasted? What is the surface area of the solid? What would the surface area be if we made it with flaps? What is the area of the flaps?

Conclusions

This circle cutter construction technique for making polyhedra should be considered by educators as a means to not only create beautiful objects but also as a way to motivate geometric problem solving.

References

[1] Forging Connections Conference website, http://forgingconnections.wordpress.com/ (as of Feb. 17, 2012)

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[4] The Mathlab.com, Making the Dice of the Gods, http://www.themathlab.com/wonders/godsdice/godsdice.htm (as of Feb. 17, 2012)

[5] Douglas B. Aichele and John Wolfe, *Geometric Structures, An Inquiry Based Approach for Prospective Elementary and Middle School Teachers*, Pearson Prentice Hall, (2008), pp. 5-6.

[6] Sándor Kabai, "Inside and Outside the Rhombic Hexecontahedron," *Proceedings of Bridges 2011*, pp. 387-394.

[7] Gerard A. Venema, *Foundations of Geometry*, (2012), Pearson.