Twisted D-Forms: Design and Construction of D-Forms with Twisted Prismatic Handles with Developable Sides

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Abstract

In this paper, we present *Twisted D-Forms* that are constructed by connecting faces of a planar polyhedra with a set of twisted prismatic handles with developable sides. We have designed and constructed a variety of twisted D-Forms using sheet metals or papers. These shapes consist of only a small number of pieces that are cut with laser cutter. Figure 1 shows an example of such twisted D-forms.

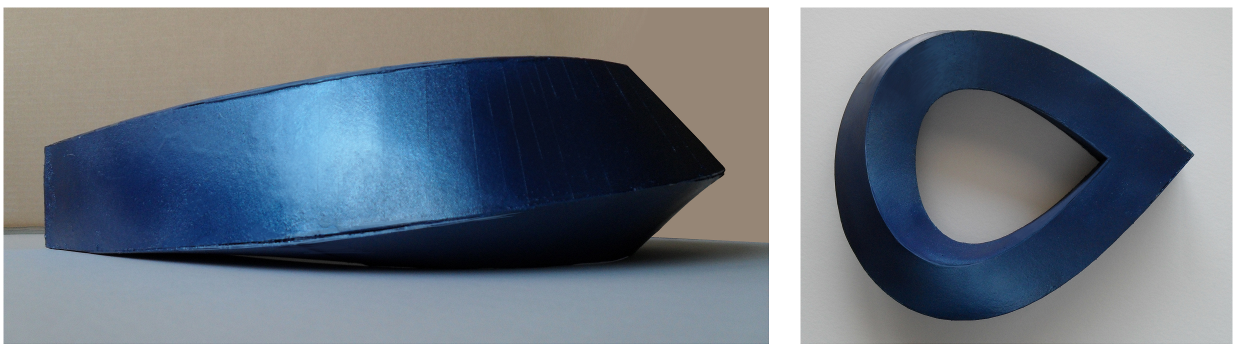

Figure 1: *An example of developable surface that is constructed from two twisted pieces of paper. We have painted the surface to obtain a shiny metallic look. This shape is constructed starting from a single cube.*

1 Introduction

The advances in computer graphics and shape modeling help fuel the imagination of contemporary architects, sculptors and designers by allowing them to design new forms in a wide variety of scales. World-renowned architectural firms such as Gehry Associates routinely design and construct buildings with unusual shapes such as the Guggenheim Museum in Bilbao. Designers like Tony Willis invent new forms. Sculptors such as Eva Hild discover and design unusual minimal surfaces. Large scale shapes such as buildings and sculptures are almost always uniquely designed and constructed. The more frequent use of unusual shapes in architecture and sculpture results in a demand for research reducing the construction cost. One possible way for cost effective construction is to use building blocks that can be produced economically and can be assembled easily. In the construction of an unusual architectural structure, it is common to use developable surfaces since they are easy to manufacture and assemble.

For the design and construction of large scale curved shapes, pieces of developable surfaces are most useful since they can be manufactured inexpensively by using laser-cutters on thin metal sheets or papers. The final shapes can be constructed by physically joining these pieces of metal sheets or papers. In mathematics, a developable surface is a surface with zero Gaussian curvature. In other words, a developable surface can be flattened onto a plane without distortion. Thin metals and paper sheets are examples of developable surfaces. In this paper, we introduce a method to design and construct shapes with twisted developable pieces. Figure 1 shows a paper prototype that is designed and constructed with our method. The whole shape consists of only two pieces of paper stripes that are cut with laser cutter.

One of the intriguing types of shapes can be obtained by twisting papers. The most well-known example of paper twisting is the Moebius strip, which can easily be obtained by half-twisting a paper and connecting ends.

For computer representation the problem is that paper twisting is a physical operation that does not correspond to mathematical twisting. Mathematical twisting does not exactly correspond to physical twisting of developable surfaces since it cannot maintain the developability of a surface. A physical twist is considered a twist because it is an approximation of a mathematical, and it can be done only if the paper strip is relatively narrow. Therefore, it is hard to design freely twisted papers using computers. Fortunately, physical twisting of a developable surface can be mathematically represented by triangle strips consisting of a large number of triangle pairs that lie head-to-toe across these developable surface strips.

Figure 2: *A genus-4 twisted D-form table stand.*

In this work, we present shapes that are constructed from a set of twisted papers, we call called *Twisted D-forms*. These shapes consists of twisted prismatic handles with developable sides. We design these handles using the handle creation tool in TopMod3D [1], which is a publicly available modeler that has been developed, implemented by our research group at Texas A&M University [2, 3]. TopMod3D is compatible with commercial modeling systems i.e. models created in this system are portable, and can be manipulated in other systems like Maya. The handle creation tool allows designing twisted handles that consists of strips of long triangles. Using this approach it is possible to design shapes of high genus. This initial triangulated model let us do minor modifications in the designs using commercial software such as Maya [4] without destroying the developable property. We unfold the model using Pepakura [5]. We constructed a large number of small scale prototypes using paper (see Figures 1 and 2). We will construct one or two of them larger scale using thin metal or plastic sheets.

2 Motivation

Developable surfaces are particularly interesting for sculptural design. It is possible to find new forms by physically constructing developable surfaces. Recently, very interesting developable sculptures, called D-forms, were invented by the London designer Tony Wills [6] and first introduced by John Sharp to the art and math community [7]. D-forms are created by joining the edges of a pair of sheet metal or paper shapes with the same perimeter [7, 8, 6].

Despite its power to construct unusual shapes easily, there are three problems with physical D-form construction. First, the physical construction is limited to only two pieces. It is hard to figure out the perimeter relationships if we try to use more than two pieces. The second problem with D-form construction is that until we finalize the physical construction of the shape we do not exactly know what kind of shape will be obtained. The third problem is that Wills' D-forms are constrained to genus-0 surfaces.

Akleman et al. [9] introduced a computation method that provides an alternative to physical D-form construction by providing a partial solution to the first two problems. Their implementation allows the user to design D-forms directly in software and their D-forms can consist of more than two pieces. One advantage of such a method is that the user can visualize the final shape before physical construction of the shape. The computer-designed D-forms can be unfolded using Pepakura, a commercially available polygonal unfolding software [5]. Once unfolded, the pieces can be cut using a laser cutter and glued together to create physical D-forms. The fundamental idea behind their computational method is to slice a planar mesh with planes. The problem with planar slice operations is that they can only allow to create convex D-forms. To create positive genus D-forms there is a need for a computational operation that can allow to change the genus of the surface.

This paper introduces an operation that can add prismatic handles with developable sides. The operation also allows to twist the handles. This approach also allows the users to design D-forms directly in computer and to visualize the final shape before physical construction of the shape. The computer-designed forms can still be unfolded, laser cut and glued to construct physical twisted D-forms.

3 Previous Work

Developable surfaces are defined as the surfaces on which the Gaussian curvature is 0 everywhere [10]. The developable surfaces are useful since they can be made out of sheet metal or paper by rolling a flat sheet of material without stretching it [3]. Most large-scale objects such as airplanes or ships are constructed using sheet metals. Since the sheet metals are easy to bend, but hard to stretch, they can easily be formed into developable surfaces.

Figure 3: *A genus-2 twisted D-form shape that is designed and constructed by our approach. This particular shape is designed starting from a single cube.*

Developable surfaces are useful in sculptural design since it is possible to find new forms by physically constructing developable surfaces with papers. Antoine Pevsner is one of the first sculptors who experiment with developable surfaces [11]. Ilhan Koman during the 1970's invented a number of developable forms [12, 13, 14, 15, 16] (see Figure 3). Sculptures of Richard Serra are also developable [17, 18]. Recently, very interesting developable sculptures, called D-forms, were invented by the London designer Tony Wills and introduced by Sharp, Pottman and Wallner [7, 19]. D-forms are created by joining the edges of a pair of sheet metal or paper with the same perimeter [7, 19]. Pottman and Wallner introduced two open questions involving D-forms [19, 20]. Sharp introduced anti-D-forms that are created by joining the holes [8]. Akleman & Gonen presented a method for computer aided design of D-forms [9]. Ron Evans invented another related developable form called Plexagons [21].

The developable surfaces are also becoming popular among contemporary architects to design new architectural forms. However, the architectural design with developable surfaces requires extensive architectural and civil engineering expertise. Large architectural firms such as Gehry Associates, Asymptote Architecture and Coop-Himmelblau can take advantage of the current graphics and modeling technology to construct such revolutionary new forms [22, 23, 10, 24].

4 Methodology

To design twisted D-forms that consist of handles that are bounded by simple developable strips, we use TopMod3D, [2, 3]. The design and construction process consists of the following steps:

1. *Start with one or more polyhedral shapes with planar or developable faces.* The requirement of planar or developable faces is necessary only in the case that one face is covered by handles as described in the next step. If an uncovered face is not planar or developable, the resulting shape may not consist of only developable parts.

2. *Connect any given two faces of the initial polyhedral shape with twisted handles.* The only constraint is that the faces to be connected must be the same type such as two triangles, two quadrilaterals or two pentagons. This requirement guarantees that we can connect the faces with prismatic handles. These handles are nothing but swept surfaces that are approximated as deformed prisms. The most crucial step is that we approximate these handles with very large number of segments using the handle creation tool in TopMod3D [1]. We also pair-wise connect the corners of the two faces in such a way that the resulting handles are twisted in space. TopMod handle creation tool provide a set of parameters. The users adjust the parameters to achieve a desired look. This procedure creates twisted handles that consists of long triangular strips.

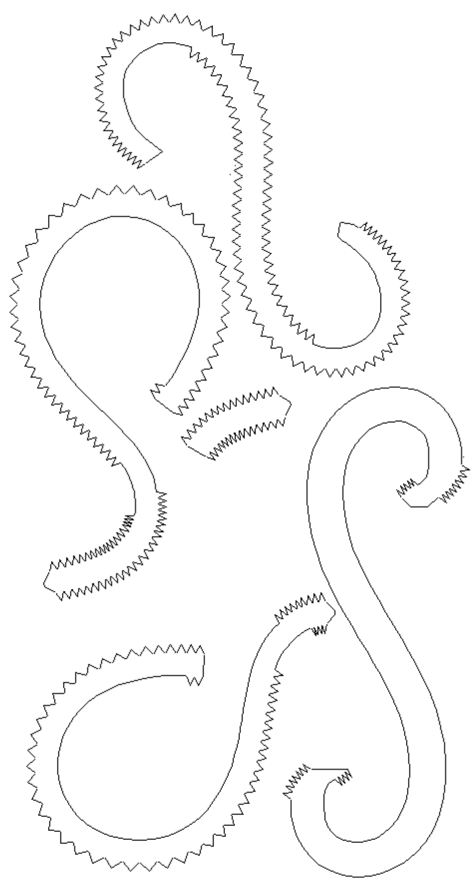

Figure 4: *An example of Pepakura unfolded twisted D-form. This particular one can even be reduced into one piece.*

3. *Continue until obtaining a desired topological structure.* We then continue the procedure until obtaining a desired topological shape. Each handle creation increases the genus of the shape by one. With this procedure one can obtain a very high genus surface with twisted handles. TopMod does not provide tools to apply some desired geometric deformations to those handles and resulting surface.

4. *Make geometric modifications to obtain a desired geometric structure.* To make geometric modifications, we export the final shape to Maya [4]. Using a wide variety of Maya tools to change geometry, we make some alterations until we obtain a desired shape. Any minor modification that maintains the basic strip quality of the faces is allowed. Such minor modifications do not destroy the developable property since final TopMod model consists of long skinny triangles.

5. *Unfold the shape.* We then export and unfold the model using Pepakura [5]. When handles are twisted, the number of unfolded pieces can be small. Pepakura does not automatically find minimal number of pieces, but, it provides user control to obtain better unfolding. With our method, unfolding gives only a small number of individual pieces, which reduces the difficulty of dealing and joining huge number of pieces. An example of unfolded pieces is shown in Figure 4.

6. *Construct the shape.* We then cut the pieces with a laser cutter and join them together simply using a glue gun. The construction process of the twisted D-form shown in Figure 1 is shown in Figure 5.

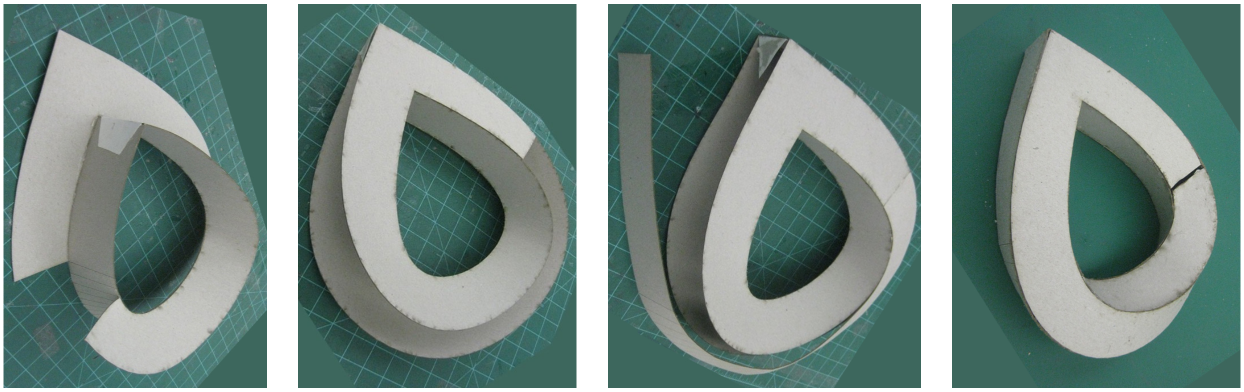

Figure 5: *The construction process of the twisted D-form shown in Figure 1.*

This procedure allows us to obtain high-genus developable shapes without worrying any constraints. These developable shapes can have any number of handles as shown in Figures 3, and 6. Since the handles are twisted, the resulting shapes provide visual puzzles, which can be perceptively challenging and interesting.

We have tested this approach in a architecture studio class and the student group, which consisted of Lauren Wiatrek, Catlan Fearon and Ronald Eckels, have easily created a significant number of twisted D-form shapes. Based on this experience, we claim that the method is easy to use and understand.

5 Conclusions and Future Work

In this paper, we presented a method to design and construct shapes with twisted D-form pieces. With this method, interesting shapes can be designed and constructed using sheet metals, plastic or paper. Using the method, we have constructed a large number of small scale prototypes using paper. These shapes consist of only a small number of pieces that are cut with laser cutter. We are currently in the process of constructing some of them in larger scale using some special plastic sheets and PVC laminated surfaces, called SINTRA.

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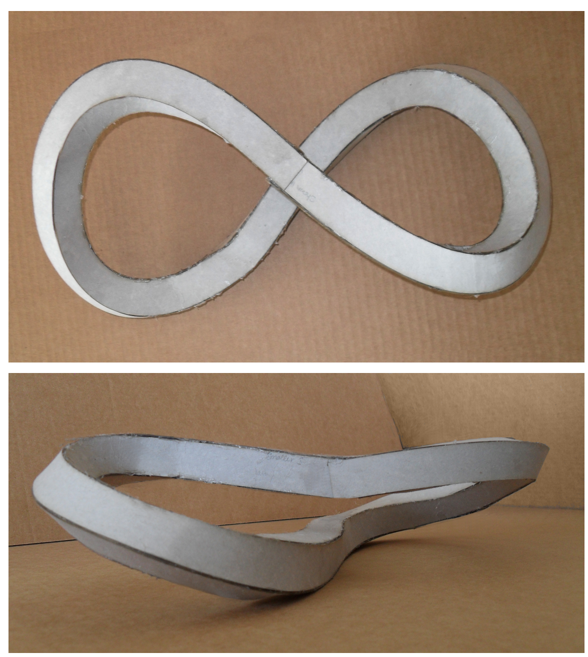

Figure 6: *A genus-1 twisted D-form that is designed and constructed from two cubes.*

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