Using Technology to Explore the Geometry of Navajo Weavings

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Abstract

The Navajo Nation is famous for the creation of many styles of weavings, with respect to variations produced by different types of yarns, geometric patterns, and pictorial themes. This article will highlight traditional and contemporary Navajo weaving styles and the use of technology to explore the fundamental regions and to find the fractals within Navajo weavings.

Introduction

The Navajo Nation is recognized for its many styles of weavings, ranging from traditional to contemporary, with new styles often emerging and evolving as individual weavers experiment with different symmetries, colors, shapes, and patterns. Professor of anthropology and curator of ethnology at the University of Arizona, Ann Hedlund notes, "Traditional Navajo narratives describe the origins of Navajo weaving in an early, deeply spiritual time. Two holy people, Spider Man and Spider Woman, brought the first weaving tools and the loom to the Navajos" [2]. Historically, it is theorized that the nearby Pueblo Indians introduced the upright loom and weaving techniques, with the Navajo weavers then developing their own styles and techniques, using wool instead of cotton, experimenting with different dyes, and ultimately creating the bold patterns that characterize the classic Chief's-style blankets. These early Navajo blankets were not just used for bedding, but also for floor coverings, clothing, and even as saddle blankets [1].

Ganado, Two Grey Hills, Storm Pattern, Chinle, and Crystal are just a few of the many styles of contemporary Navajo rugs. Often grouped into regional and thematic rug styles [1], contemporary styles are either named by the region where the style originated or by the theme of the rug's pictorial style. A regional style originally centered around a community's trading post. A rug would be sold by the trader, who often made suggestions regarding the type and color of yarn as well as the overall geometric pattern that the trader believed to be the most marketable [2]. Specific regional styles are often associated with vegetal, aniline, or undyed yarn, having geometric or pictorial motifs, as well as certain types of layouts of the design, but there are no rigid definitions to the regional styles. Individual weavers, working with the same regional style, will produce unique variations within that style [2]. The Storm Pattern (see Figure 1) is unusual in that in 1911, trader J.B. Moore of the Crystal Trading Post claimed that this pattern symbolized "Navajo mythology." There are differing scholastic opinions on this claim, with some believing that the Storm Pattern does indeed have elements of the four sacred mountains and the center of the universe, but many believe the alleged symbolism was an invention of J. B. Moore's imagination [2].

Pictorial styles can be scenic, secular, or sacred. Scenic rugs may contain animals, plants, mountains, mesas, or other elements of Navajo life. Secular rugs may have holiday symbols, historical scenes, patriotic images, or other themes. The most controversial style involves the use of sacred images from Navajo religious ceremonies, depicting sandpaintings and Navajo holy people. "Most Navajos agree that reproducing such designs is personally dangerous" [2]. Many weavers of this style "undergo ritual protection through a traditional Navajo medicine man" [2], to insure their spiritual well-being.

Figure 1: *Jason Harvey, Detail of Storm Pattern Tapestry (1995)*
Courtesy of the Arizona State Museum, University of Arizona

Figure 2: *Barbara Teller Ornelas, Detail of Two Grey Hills Tapestry (1995)*
Courtesy of the Arizona State Museum, University of Arizona

The Fundamental Region

Barbara Teller Ornelas, a Navajo weaver well-known for her Two Grey Hills (see Figure 2) style of weaving, when asked how she designs the patterns in her rugs, responded, "It's like sitting inside a puzzle. The rug is symmetrical both horizontally and vertically. A rug is like a kaleidoscope, you only need to design one quarter of the rug, and then duplicate that pattern" [1]. This ¼ of the rug, the smallest region that can be used generate the entire pattern of the rug, is called the fundamental region. A study of fundamental regions, in Navajo rugs, offers mathematics students an excellent opportunity to explore transformational geometry. Although many Navajo weaving styles have a fundamental region of ¼, students can search online for weavings with smaller fundamental regions, or experiment with color changes in an existing pattern, where a weaving could have a fundamental region of 1/16 or even as small as 1/20 [3].

The Arizona State Museum of the University of Arizona has an online Navajo Weaving Exhibition with exciting activities that demonstrate how a fundamental region of ¼ generates an entire rug. In the first activity (see Figure 3), examples of existing rugs are given, where the fundamental region is reflected horizontally and vertically to produce the entire rug. The second activity (see Figure 4) allows a student to design their own fundamental region, using different shapes, with adjustable sizes and colors. When the region is completed, the region is reflected horizontally and vertically, creating an original design [1]. This online weaving exhibition not only provides a virtual way for students understand the power of the fundamental region and experience the thrill of designing their own rug; it also contains extensive information about the history of Navajo weaving, the many different styles, and how a rug is created from the shearing of wool off a sheep all the way to the actual weaving of the rug on a loom.

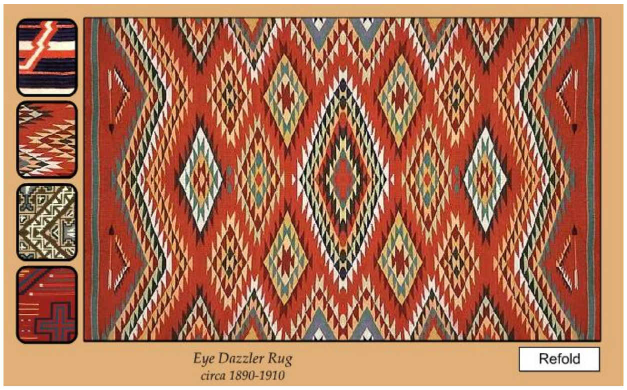

Figure 3: *First activity with examples of existing patterns. Courtesy of the Arizona State Museum, University of Arizona*

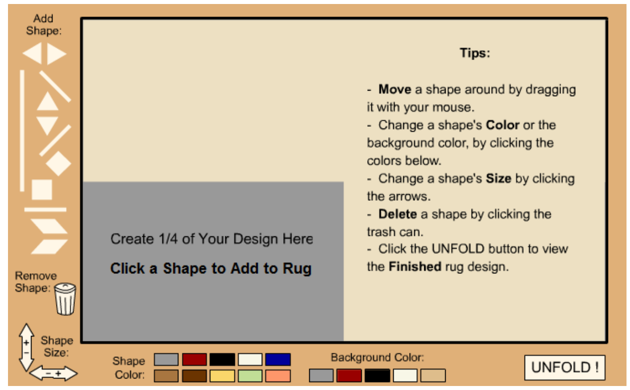

Figure 4: *Second activity of designing an original pattern. Courtesy of the Arizona State Museum, University of Arizona*

Exploring the Fractal from the Ganado Round Rug

The Ganado style rug (see Figure 5) provides inspiration for students to combine technology, creativity, and mathematics, through the use of the Geometer's Sketchpad (GSP), their personal artistic talents, and their knowledge of fractals and fundamental regions. Since not all students may be familiar with fractals, it may be helpful to note that a fractal typically has three characteristics. They are visually self-similar, produced by iteration and they may have a non-integer dimension. Examples of fractals found in nature, such as ferns and tree branches, may help students to understand the concept of self-similarity, where the smaller part of an object looks like the larger part, but on a smaller scale [5]. Iteration is best explained as a repeated process. The concept of fractal dimension is quite complicated and advanced students could be advised to explore this concept on the Internet by conducting a search for Hausdorff Dimension.

Figure 5: *Detail of a round rug with Ganado style; Photograph courtesy of Hubbell Trading Post National Historic Site.*

In this activity, students will first sketch fundamental regions of ¼ and then using horizontal and vertical reflections, design fractals that are not only used in this Ganado Round Rug, but often seen in many other Navajo rugs. The most difficult part is sketching the outline for the fundamental region. There is more than one way to do this and frequent users of the Geometer's Sketchpad may actually enjoy the challenge of finding their own method of designing the fundamental region. For occasional users of GSP, some extra support may be warranted. The GSP Help menu will provide detailed instruction for any feature in the Geometer's Sketchpad. Listed below is one possible method of sketching the fundamental region.

To begin our sketch of the fundamental region, we initially construct a parallelogram in a certain way to insure that the fundamental region reflects as desired. First construct $\overrightarrow{AX}$ and then rotate X 30° (or any other acute angle you prefer) about A. Then construct a ray through this rotation point, labeled Y, using A as the endpoint. Now select an arbitrary point on $\overrightarrow{AY}$ and label it B. Construct a line through B that is perpendicular to $\overrightarrow{AX}$. Label the intersection of the perpendicular line and $\overrightarrow{AX}$ as M. Reflect A about the perpendicular line and label the reflection point as D. Construct a line parallel to $\overrightarrow{AB}$ through D. Now construct a line parallel to $\overrightarrow{AD}$ through B. Find the intersection point of these two parallel lines and label that point C. Hide everything except for the parallelogram $ABCD$ and then mark a vector from A to D and translate the parallelogram three times (see Figure 6). You may want to shrink the size of the parallelogram to make it more manageable.

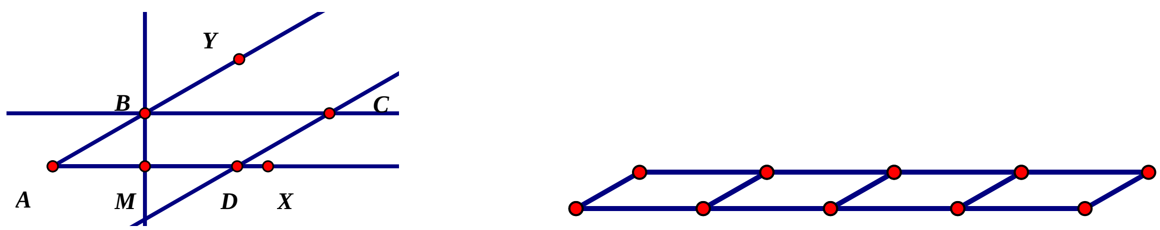

Figure 6: *The initial parallelogram construction and its translation*

Using the four parallelograms, mark a vector from D to B and translate eight times. Then depending on where you place a vertical line, you can create two different fractals (see Figure 7). Then remove all extraneous points and line segments to create the desired fundamental region. Color in the polygons, reflect the regions horizontally and vertically, and see the two fractals emerge (see Figures 8 and 9).

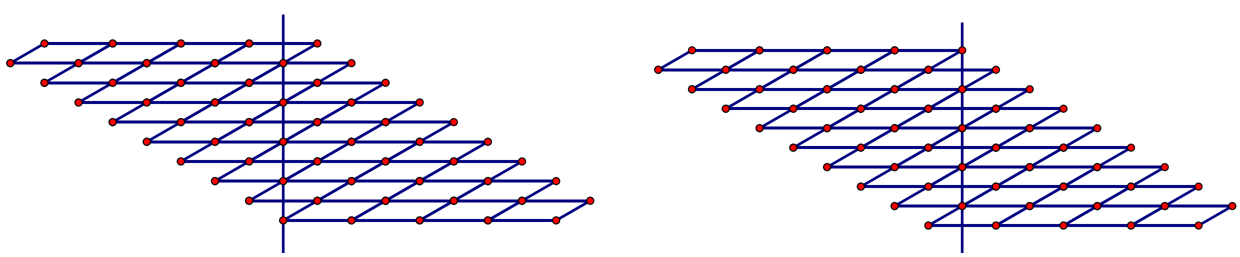

Figure 7: *The change in the position of the vertical line changes the fractal*

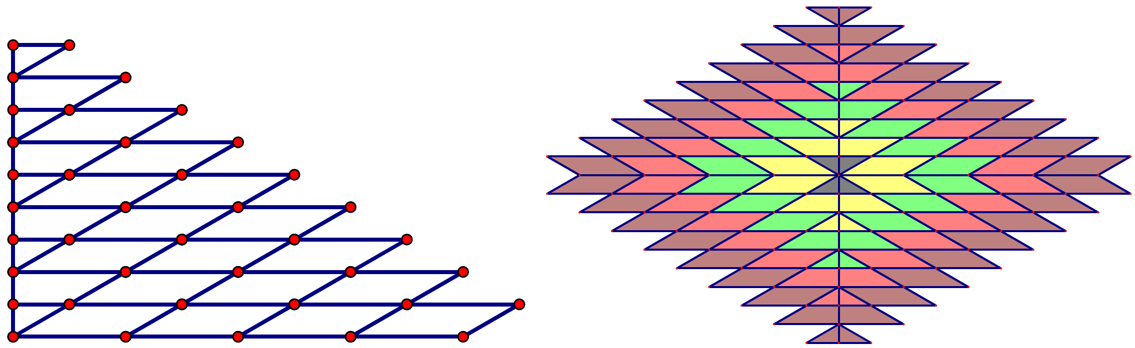

Figure 8: *The fundamental region has colors added to the interiors of the polygons and then is reflected horizontally and vertically*

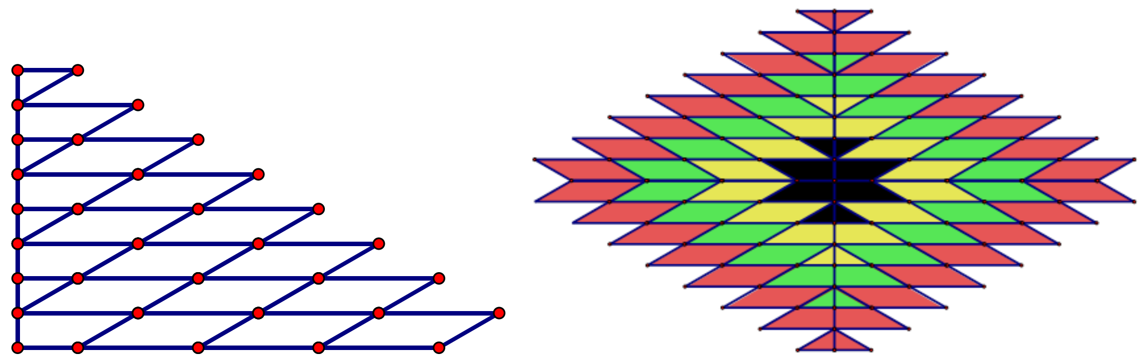

Figure 9: *A different fundamental region yields a different fractal*

A Sierpinski Gasket Inspired by Navajo Weavings

Fans of fractals will recognize the potential to connect the Sierpinski Gasket with the traditional diamond shape found in the center of many styles of Navajo rugs. The Burntwater example, woven by Roselyn Begay (see Figure 11), provides the inspiration for the following Sierpinski Gasket design (see Figure 12). Greater detail constructing the Sierpinski Gasket can be found in the reference section [5]. Essentially, first make a right Sierpinski Gasket and save as a Custom Tool (see Figure 10).

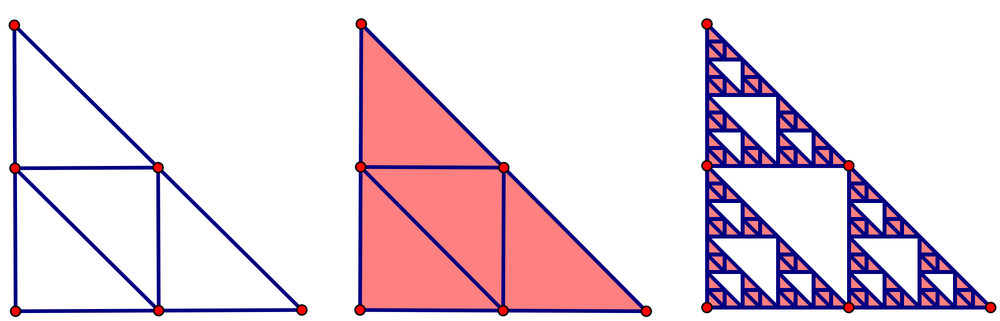

Figure 10: *Creating the fundamental region of the Sierpinski Gasket*

Now make any square, divide it into four parts, put a copy of this gasket into each of the newly created small squares, and then horizontally stretch the square, into a rectangle (see Figure 12).

Figure 11: *Roselyn Begay, Burntwater style; Courtesy of the Kirchner Collection*

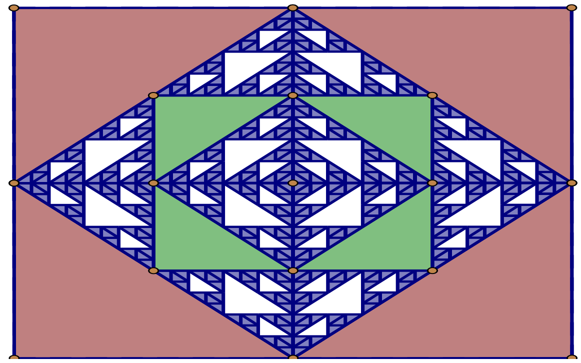

Figure 12: *Sierpinski Gasket, inspired by the Begay Burntwater rug*

Generating a Cornstalk Design and Discovering a Numeric Pattern Within

With the Geometer's Sketchpad, another pattern can be generated using a design inspired from the leaves of the cornstalk in the raised outline rug depicted in Figure 13. For this we need to study the iteration of geometric objects in Sketchpad using finite independent points (Please see [5] for detailed instructions). A polygon can be sketched that imitates two of the leaves, in the bottom of the corn stalk, such as the polygon in Figure 14.a.

Figure 13: *Larry Yazzie, Diné (Navajo) Detail of Raised Outline Rug (1994) Courtesy of the National Museum the American Indian, Smithsonian Institution [Catalog Number: 25/4375].*

In order to comfortably generate the design, we need to make this polygon with only two independent points, *A* and *B*. For this, draw $\overline{AB}$ in *Sketchpad*. Find the midpoint of $\overline{AB}$ and label it *O*. Using $\overline{AO}$ and $\overline{OB}$, construct equilateral triangles above $\overline{AB}$, as well as the midpoints of $\overline{AO}$ and $\overline{OB}$. Construct the three equilateral triangles below $\overline{AB}$, as shown in Figure 14.a. Also using the midpoints of the interior sides of the two equilateral triangles above $\overline{AB}$, construct the smaller equilateral triangle, with point *D*, at the top of the triangle, as shown in Figure 14.a. Now hide all segments but the sides of the cornstalk leaf.

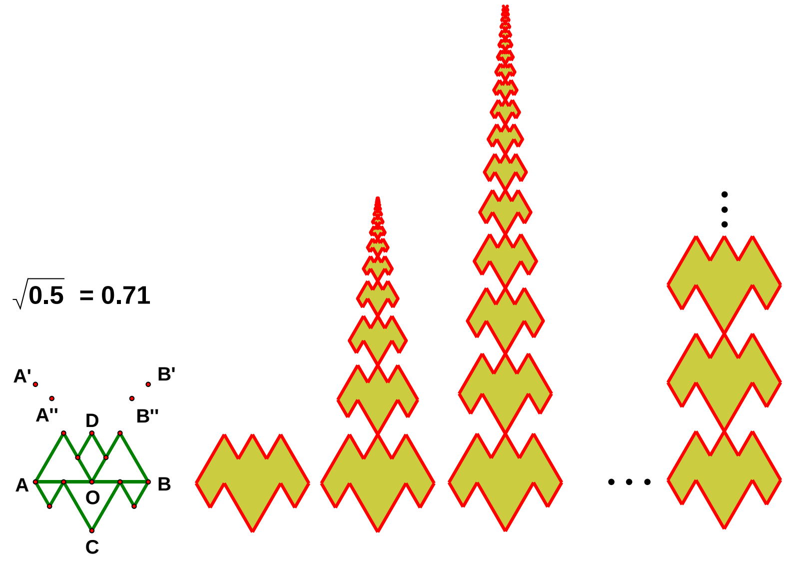

Figure 14: *Design inspired by the cornstalk leaves in the raised outline rug from Figure 13.*

To generate each cornstalk design first translate the shape using vector C to D to see where A and B go (in fact we only need to find the new location of A and B and call these A' and B'). Consider that we want to create one branch so that the area of each new set of cornstalk leaves is $(n - 1)/n$ smaller than the previous set, $n \geq 1$. For this, we make D the center and dilate A' and B' with the factor of $\sqrt{(n-1)/n}$ to find A'' and B''. Now hide A' and B'. Select A and B and then iterate by going from A to A'' and B to B''.

It is important to notice that if we use k as the scale factor of similar shapes, then the scale factor for their areas will be k^2. Therefore to have a similar shape with an area $(n - 1)/n$ smaller than the previous one, we need to use $k = \sqrt{(n-1)/n}$ but we first need to calculate this number in Sketchpad as a decimal and then use it. If we consider Figure 14.b to have an area of 1 square unit, then Figure 14.c has an area of $1 + a + a^2 + a^3 + a^4 + \ldots$ where $a = ½$. Since the series converges, the area can be represented by the sum of an infinite geometric series formula, $1/(1 - a)$, which means the area equals 2 square units. In Figure 14.d where $a = 2/3$, the sum of the series is 3. In the fourth series, where $a = ¾$, the sum is 4. We note that $(n – 1)/n$ converges to 1 when $n \to \infty$. Figure 14.e presents the series of $1 + 1 + 1 + 1 + \ldots$ as $n \to \infty$. Analyzing this pattern, as shown in Figure 15 below, we discover that within these very complicated iterations, we find none other than the very simple set of Natural Numbers.

First Step A_1	**Second Step** A_2	**Third Step** A_3	……….	n^{th} **Step** A_n
1	**1 + 1/2 + 1/4 + 1/8 + …**	**1 + 2/3 + 4/9 + 8/27 + …**	⟶	**1 + 1 + 1 + 1 + …**
1	**2**	**3**	⟶	∞

Figure 15: *The area of the design inspired by the cornstalk leaves.*

Conclusion

Using technology to explore the geometry of Navajo weavings, offers students the opportunity to expand their cultural horizons, hone their technological skills, and increase their understanding of mathematics. As students experience the satisfaction of creating their own simple geometric designs, they will ultimately appreciate the inspiring complexity and beauty of Navajo weavings.

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