The Art and Mathematics of Tangrams

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Abstract

This article is about the application of tangrams in teaching mathematical concepts. A brief history of the tangram is introduced, and is followed by a series of examples that involve arranging a set of thirteen convex polygons. These examples help to illustrate not only the mathematical elegance of tangrams, but also serve as a springboard for discussing the tangram in actual educational contexts.

Introduction

"Give me a fulcrum," Archimedes is reported to have said, "and a place to stand—and I will move the world". Archimedes' famous words have piqued the interests of thousands in the field of physics. Future educators may be coin a similar phrase. "Give me a set of Tangrams, and I will reveal the image of the world with geometry". The tangram is a traditional Chinese game, which has been popular among people for thousands of years in both Eastern and Western cultures. As simple as they appear, Tangrams may serve as excellent illustrations of some the inherently interesting nature of geometry. Due to their hands-on nature, they serve as good learning tools for students due to their ease of being physically and easily manipulated.

History of Tangram

The tangram is one of the oldest Chinese puzzles dating from hundreds of years ago. It is also called Qiqiaoban in Chinese, literally translating to "seven boards of skill". It is a dissection puzzle consisting of seven flat shapes, called *tans*, which are put together to form shapes. The objective of the puzzle is to form a specific shape (given only in outline or silhouette) using all seven pieces, which may not overlap (Figure1).

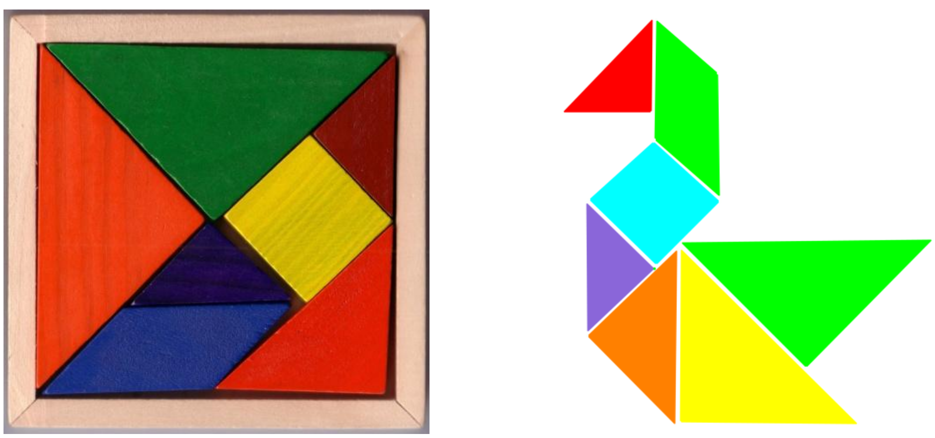

Figure 1: (a) *A set of tangram*, (b) *A tangram swan*

According to the introduction from a Chinese encyclopedia, the tangram may have roots in the *Yanjitu* furniture set of the song dynasty contributed by Siming Huang. This furniture set saw some variation during the Ming dynasty. Cheng Yan, the author of *Diejipu,* transformed the square set into triangular one, and divided it into thirteen triangular pieces to make it a butterfly shape, which was named *Diechiji*. Later it became a set of wooden blocks for playing and creating beautiful images. The images that appear all over from the wall as an interactive tool for creating artworks in classroom [1], to the artwork on the shirts [2] (Figure 2). While the tangram is often said to be ancient, the earliest known printed reference to tangrams appears in a Chinese book dated 1813, which was probably written during the reign of the Jiaqing Emperor.

Figure 2: (a) *Tangram Wall Graphics, a new interactive wall art from*, (b) *Tangram bunny shirt*

The tangram's existence in the West has been verified to no earlier than the early 19th century, when they were brought to America on Chinese and American ships. The earliest known example, given to the son of an American ship owner in 1802, is made of ivory, and has a silk box.

The word *tangram* was first used in 1848 by Thomas Hill, then President of Harvard University, in his pamphlet *Puzzles to Teach Geometry*. The author and mathematician Lewis Carroll reputedly was a tangram enthusiast and owned a Chinese book with tissue-thin pages containing 323 tangram designs. Napoleon is said to have owned a tangram set and Chinese problem and solution books during his exile on the island of St. Helena, although this has been contested by Ronald C. Read [3][4].

Tangram Mathematics in the Classroom

Small and simple in nature and by design, tangrams inherently imply a multitude of mathematical concepts. Some are easy to discover. Others, however, show more of an implicit learning style. Mathematicians have been performing a lot of research on tangrams, while in most contexts, they are regarded as "entertainment mathematics" due to their accessibility as well as their workability in the daily learning of mathematics. All of these features enable them to become the part of geometry class for students in different levels.

How many convex polygons can you make with the tangram? This is a question brought up by Japanese mathematicians in the 1930s. The question sparked a lot of interest among mathematicians. Finally, this problem was solved by two scholars from Zhejiang University. Their discourse "A Theorem on the tangram" published in the "The American Mathematical Monthly" presented a brief yet accurate proof. According to their proof, there are only 13 convex polygons can be formed by the tangram. Figure 4 shows 12 of them, the other one is the square set from which the tangram was divided[5][6].

Figure 3: *12 other convex figures made from tangram set besides square*

In the geometry class when the concept "convex polygon" is introduced, teachers can ask students to tell whether the seven tans in the tangram are convex polygon or not. Furthermore, teachers may allow their students construct convex polygons with tangrams based on their knowledge of what a convex polygon actually is. This is an excellent group work activity. Students discuss the shapes that they construct and share their opinions on whether the figures are convex polygons or not. With their cooperation and teachers' instruction, students will eventually work out the 12 figures shown in Figure 3 plus the original tangram set, which is a square and is also regarded as a polygon (see Figure 1.a).

In addition, the most important function of this activity is to enable students to reinforce their ability to decompose a figure with shapes given in a tangram set. Level 1 figures are common polygons that students see frequently in geometry classes. Thus, based on the given tan shapes, they can easily discover their structures, which are composed of a square and the arrangement of triangles. However, level 2 figures reveal some hidden challenges for students, since those structures are not that easily discovered. However, it is still possible for students to make hypotheses about their structures. Along the same lines, when it comes to the level three figures, the possible structures are almost hidden. Especially when the number of sides is increased, constructing these figures with tangrams becomes very challenging for students. However, if they work together and try to analyze the workable relations between each side, they may pleasantly surprise themselves with the solution.

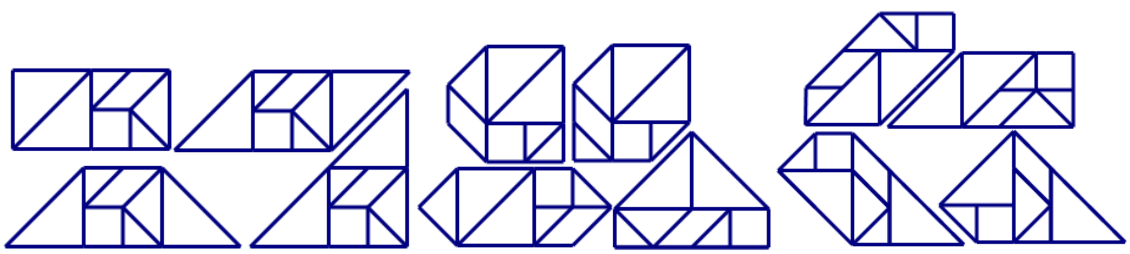

Figure 4: (a) *Level 1 figures,* (b) *Level 2 figures,* (c) *Level 3 figures*

Instructional Strategies that Help Apply Convex Polygon Construction

(1) Level 1 figures could be used to teach the concepts of basic polygons. Those four shapes are the most commonly-used and discussed polygons with properties that students should know. By asking students to construct those figures with tangrams, students will discover that those figures can be constructed with a square as a part and different arrangements of triangles. Thus, students would have a basic idea of decomposition of the figures, which would be a great help for the

high-level study of geometry. Actually, figures at each level have a "major" figure that are composed of part of the figures. The "Major parts" are highlighted below:

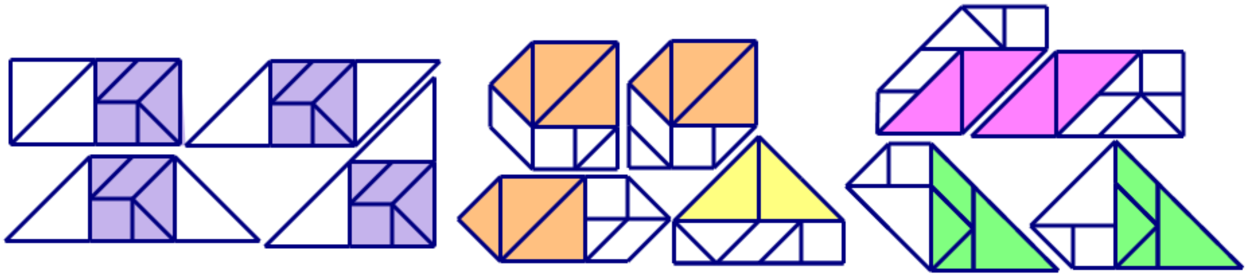

Figure 5: *Figures with "major" part highlighted.*

(2) Level 2 figures are more complicated than those of level 1, which are not as easy to decompose as level 1 figures. However, some of the figures are just different by a move of a piece of tan. For example, Figure 6 (a) illustrates the movement.

But why it can be a good fit? As the length relations show, the length of hypotenuse is the same as the long side of the parallelogram.

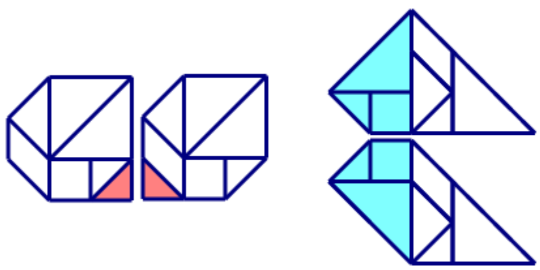

Figure 6: (a) *The difference of the two figures*, (b) *The reflection in level 3 figures*

(3) Level 3 figures are the most complicated ones, which most students would have difficulty composing them. The idea of reflection would be a good concept to teach with part of the two of the figures (Figure 6.b.).

References:

[1] Wall Tangram http://blog.walls360.com/yiying-wall-tangrams/

[2] Tangram Bunny Shirt http://www.zazzle.com/tangram_bunny_tshirt-235727522368814732

[3] Tangram. http://en.wikipedia.org/wiki/Tangram

[4] Slocum, Jerry (2003). *The Tangram Book*. Sterling.

[5] Fu Traing Wang; Chuan-Chih Hsiung (November 1942). "A Theorem on the Tangram". *The American Mathematical Monthly* 49 (9): 596–599.

[6] Zhang jingzhong. "Qiqiaoban, Jiulianhuan, Huarongdao, the three Chinese ancient games"