A *Mathematica* GUI for Generating Conway Tiles

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Abstract

The *Conway Tiling Criterion* specifies conditions sufficient to guarantee a geometric shape will periodically tile the plane. We present a suite of dynamically interactive software tools that facilitate the creation of these Conway tiles. The variety of tiles that can be produced is astounding, and the resulting tiles can be exported to a variety of image formats, individually or as completely tiled regions comprising multiple tiles. The tools are freely available [2], but require the *Mathematica* software system.

The Conway Criterion

The *Conway Tiling Criterion* is a simple set of conditions, sufficient to guarantee that a compact two-dimensional geometric shape will tile the plane. A full description and explanation of the criterion is given in [1], but we provide a brief summary here.

Let T be a topological disk whose boundary is a simple closed curve. Suppose there are six successive points A, B, C, D, E, and F on the boundary of T, so that:

1. The portion of the boundary from A to B is congruent by translation to the portion of the boundary from E to D.
2. The remaining four boundary segments are each *centro-symmetric*. That is, each is congruent to itself when rotated a half-turn about its midpoint.

Then T will tile the plane, and moreover a tiling can be generated by rotating T through the midpoints of its four centro-symmetric "sides." It is permissible that two or more of the six points A–F may coincide. For example if $A = B$ and $D = E$, then we have a four-sided figure with four centro-symmetric sides.

We call such shapes *Conway tiles* in honor of John H. Conway, who developed this criterion. Below we see a tiling similar to M.C. Escher's symmetry drawing no. 107, created with this software.

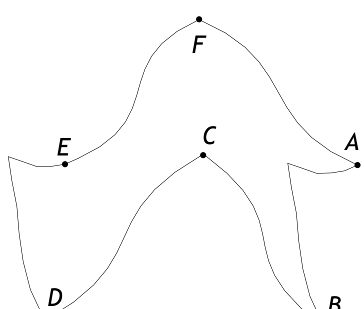

Figure 1 : *A Conway tile*

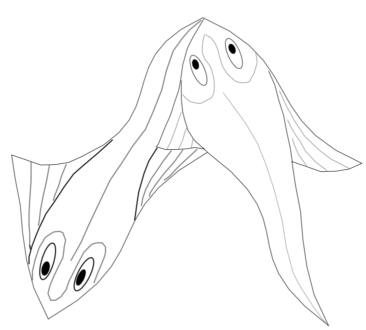

Figure 2 : *The same tile, decorated*

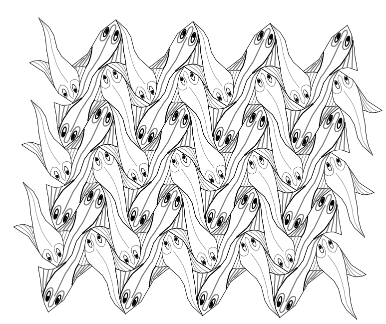

Figure 3 : *The resulting tiling*

Creating the Boundary Segments

The software allows one to individually create five of the six boundary segments (ED is just a translation of AB, so there are only five). Each boundary segment is created via a simple point and click interface. The user places a sequence of "locators" in an image pane (see Figure 4), and decides on one of two options: a spline curve based on the locators, or a piecewise linear curve with segments placed between them. Once added, individual locators can be moved via a simple drag, or deleted. When a centro-symmetric boundary segment is created, the user creates only half of the curve—from one endpoint to the midpoint, as described above. The centro-symmetric condition is applied automatically to create the full boundary segment. See figures 4 and 5 for an example.

After a boundary segment is completed, it is exported to a second pane, which shows the entire tile (see Figure 5). The user may move the positions of five of the six points (condition 1 of the Conway criterion removes a degree of freedom), being careful to ensure that the boundary of the tile is a simple closed curve. The individual boundary segments may be edited and replaced as often as necessary. The finished tile may then be decorated with freehand drawing tools, and finally rendered alone or as a completely tiled region (see Figure 6).

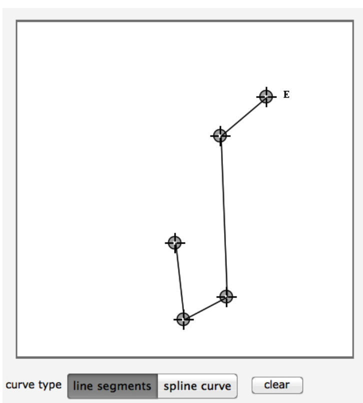

Figure 4: *Boundary segment EF, shown from E to midpoint*

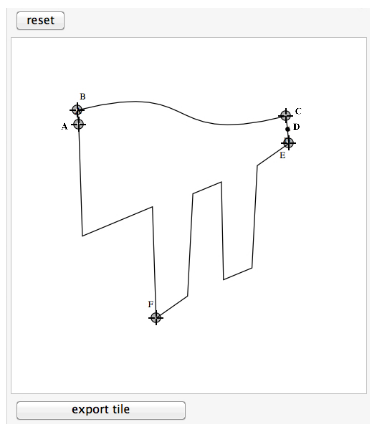

Figure 5: *A tile with this boundary segment*

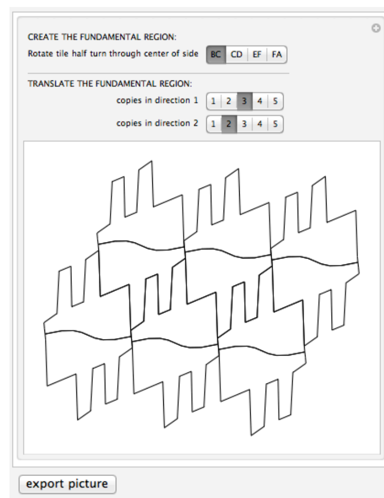

Figure 6: *The resulting tiling*

The process of decorating a tile can be done within *Mathematica*, using its built-in drawing tools, before the final step. This means that decorating the interior of a tile is done just once, for a single tile. The tile or a completely tiled region may then be exported to a variety of image formats. Of course, alternating colors and other effects may be added after export, using any image editing software the user desires.

References

[1] D. Schattschneider, "Will it tile? Try the Conway Criterion!," *Mathematics Magazine*, **53** (4), Sept. 1980, 224–233.

[2] B. Torrence, Conway Tile Builder, `http://faculty.rmc.edu/btorrenc/tilebuilder/`.