

Tiling and Weaving with Permutation Functions

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Abstract

This article shows how permutation functions and Artin's Braids are related and shows how the combination of the two can be utilized to create artistic and practical applications.

Permutation Functions and Artin's Braids

In this article, the six permutation functions defined on $S = \{A, B, C\}$ will be designated as I, f, g, h, F, and G. Each is a bijection function of $\{A, B, C\}$ onto itself. To represent these functions and their compositions, a form of "braids" can be employed. It is a technique by Emil Artin, a famous Austrian/American mathematician, who lived from 1898 to 1962. In Figure 1, below, there are portions of six braids which match the functions. Upon studying the braids, it should become clear how the matching is made. One braid shows function g, where $g(A) = C$, $g(B) = B$, and $g(C) = A$. Another shows $G(A) = C$, $G(B) = A$, and $G(C) = B$. The rest of the braids in Figure 1 represent the remaining permutation functions.

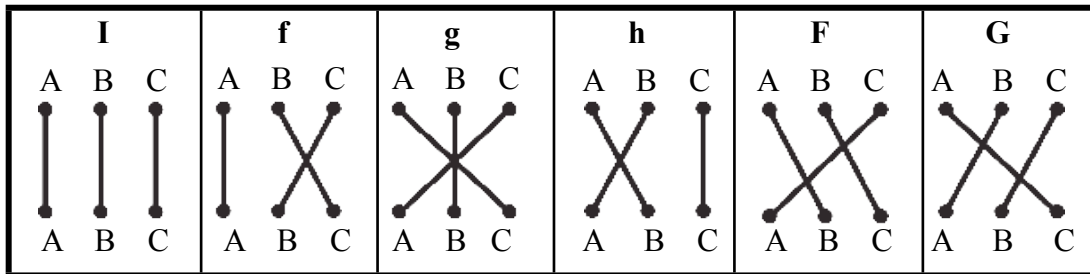


Figure 1

A Symmetric Group of Braids

The symmetric group of the six permutation functions from Figure 1 along with composition "o" is presented in the Cayley Table 1. Because of the relationship between the permutation functions and the braids, the *body* of Table 1 can be replaced by Figure 2. It might appear to be flawed, lack artistic beauty, or even be ugly.

Art Created from a Symmetric Group

It has been said that some weavers of beautiful Persian carpets introduce an intentional flaw because only the almighty is "perfect". The flaw becomes imperceptible when the entire carpet is seen. It might be said

that, “you can’t see the trees for the forest” . The symmetric group is a good example of a perfect mathematical structure which some laypeople might think has a minor flaw because “o” is not commutative. However, it too can be used to design a carpet or to create a mosaic If Figure 2 is reduced in size, is replicated, and the replications are connected, a herringbone design for weaving a carpet results as illustrated in Figure 3. The perceived “flaw” of Figure 2 becomes imperceptible. Likewise, adding shading to Figure 2 produces Figure 4 which resembles a design for a tile mosaic which seems to reflect beauty. Thus, out of an interpretation of a symmetric group artistic and practical applications can be created.

First Function Performed

| o | I | f | g | h | F | G |
|---|---|---|---|---|---|---|
| I | I | f | g | h | F | G |
| f | f | I | F | G | g | h |
| g | g | G | I | F | h | f |
| h | h | F | G | I | f | g |
| F | F | h | f | g | G | I |
| G | G | g | h | f | I | F |

Table 1

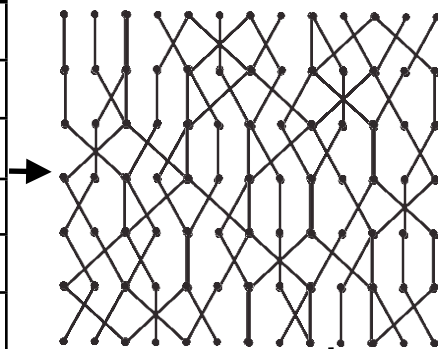


Figure 2

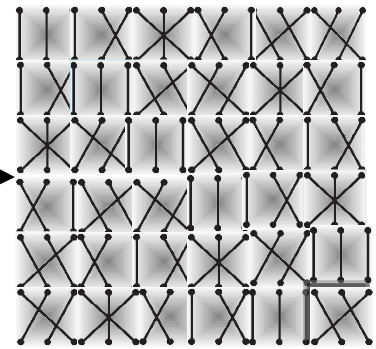


Figure 4

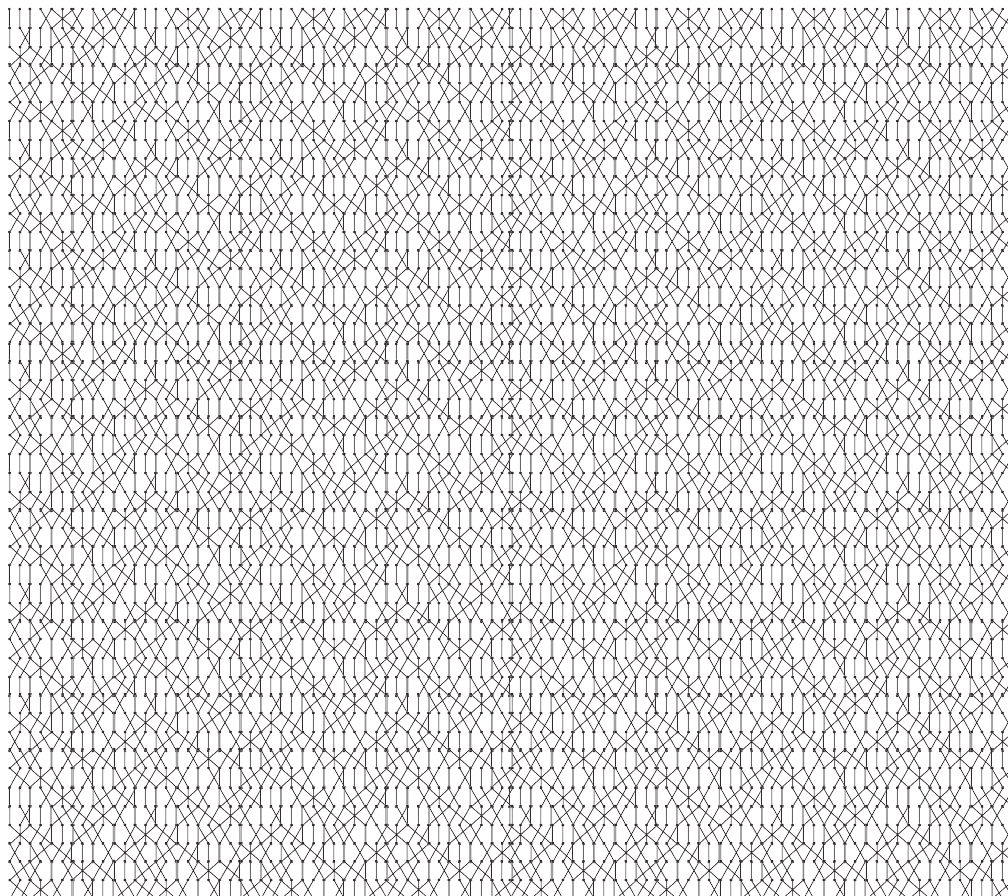


Figure 3